EUROPEAN JOURNAL OF STEM EDUCATION

European Journal of STEM Education,
2026, 11(1), 15
eISSN: 2468-4368

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Guided Inquiry-Based Instruction Assisted by Variation Theory as a Strategy to Enhance Students' Achievement and Goal Orientations in Solid Geometry

Abebaw Yeshanew [1*], Tesfu Belachew [2], Habtamu Gezahegn [1], Tadele Tesfa [1]

[1] *Hawassa University,* ETHIOPIA
[2] *Madda Walabu University,* ETHIOPIA

***Corresponding Author:** y.abebaw2012@gmail.com

Citation: Yeshanew, A., Belachew, T., Gezahegn, H., & Tesfa, T. (2026). Guided inquiry-based instruction assisted by variation theory as a strategy to enhance students' achievement and goal orientations in solid geometry. *European Journal of STEM Education, 11*(1), 15. https://doi.org/10.20897/ejsteme/18145

Published: March 17, 2026

ABSTRACT

This quasi-experimental study investigated the effects of Guided Inquiry-Based Instruction (GIBI) assisted by Variation Theory (GIBI-VT) on the achievement and goal orientations of grade ten students in solid geometry. Participants (N=102) from Debre Tabor City, Ethiopia, were randomly assigned to: Experimental Group 1 (EG1, n = 31) receiving GIBI-VT, Experimental Group 2 (EG2, n = 39) receiving GIBI alone, and a Control Group (CG, n = 32) receiving the traditional teaching method (TTM). Quantitative data were collected using a validated mathematics achievement test and goal orientation questionnaire. The ANCOVA revealed statistically significant differences in mathematics achievement post-test scores. Additionally, the pairwise comparisons showed that both EG1 and EG2 significantly outperformed the CG. Moreover, MANOVA indicated EG1 scored significantly higher on mastery goals orientation than EG2 and CG. Finally, the regression analysis model revealed that student's gender, Performance-Approach Post-Test Scores (PApPostTS), GIBI-VT vs. TTM, and GIBI alone vs. TTM significantly explained 47.0% of the variance in their Mathematics Achievement Post-Test Scores (MAPostTS). The GIBI-VT is an effective strategy for enhancing students' achievement and mastery goals orientation in solid geometry. The study recommends incorporating GIBI-VT into teacher training and curricula to address persistent challenges in geometry learning in Ethiopia and similar contexts.

Keywords: guided inquiry-based instruction, variation theory, solid geometry, mathematics achievement, goal orientations

Solid geometry is a fundamental pillar of school mathematics education, essential for developing students' spatial reasoning and logical thinking skills (Jones, 2020). However, this domain poses a significant and persistent challenge for students globally, with particularly acute achievement gaps observed in Ethiopian secondary schools. The national assessments reveal that grade ten students' achievement in mathematics, especially in geometry, falls dramatically below national standards, with a mean score of 33.59 (SD = 15.10) (NEAEA, 2017).

This is attributed in the literature to the predominance of Traditional Teaching Methods (TTM), which are characterized by rote memorization and procedural practice, which may not foster the conceptual understanding and higher-order thinking required for mastering abstract geometric concepts (Etang & Regidor, 2022; Yaki, 2022; Yeshitila & Mamo, 2019). In response to these challenges, Guided Inquiry-Based Instruction (GIBI) is noted as a pedagogical approach that engages students in active exploration and problem-solving, with studies reporting associations with improved academic outcomes (Jumantini et al., 2021; Ogunjimi & Gbadeyanka, 2023).

Additionally, variation theory provides a robust framework for designing instruction that systematically helps students discern the critical features of mathematical concepts through strategic patterns of variation and invariance (Kullberg et al., 2017; Gu et al., 2017). The integration of these approaches holds significant potential; GIBI creates an environment for deep exploration, while variation theory structures that exploration to maximize conceptual discernment (Kassa & Ding, 2019; Handy, 2021; Pang et al., 2017).

Thus, this study addresses three key gaps: the combined use of GIBI and variation theory remains under-researched, especially in African contexts; few studies examine how such integrated pedagogies affect students' goal orientations alongside mathematics achievement; and limited attention given by existing research to the role of students' motivational factors, such as goal orientations, which are powerful predictors of mathematics achievement (Guo & Hu, 2021; Putwain et al., 2018) within such innovative teaching interventions. By comparing GIBI-VT with GIBI alone and TTM, the findings would provide practical insights for improving mathematics pedagogy and inform future curriculum development in Ethiopia and similar educational contexts globally.

Research questions

This study is guided by the following research questions.

1. After accounting for pre-test scores, do experimental and control groups show statistically significant differences in mathematics achievement post-test results?
2. Is there a significant difference in students' goal orientation sub-constructs post-test scores between the experimental and control groups?
3. What are the contributions of students' gender, their parents' residence (Urban vs. Rural), teaching methods (GIBI-VT vs. TTM, and GIBI alone vs. TTM), and goal orientation sub-constructs on their achievement in solid geometry?

RELATED LITERATURE REVIEW

GIBI and learning mathematics

Educators are seeking ways to prepare students for living and working in the 21st-century, ever-changing environment. In this context, mathematics education isn't only aimed at supporting students' learning of algorithms and procedural fluency but also needs to address competencies such as creativity, making connections, critical thinking, collaboration, and communication (Eurydice, 2022; OECD, 2018). In this context, Inquiry-Based Mathematics Teaching (IBMT) has been promoted as a key pedagogy to better prepare students for a dynamic society (Doorman, 2017).

Guided Inquiry-Based Instruction (GIBI), a structured form of IBMT rooted in constructivist theory, is a student-centered approach where teachers guide students to actively explore mathematical concepts, formulate hypotheses, and solve problems collaboratively (Istikomah et al., 2022). In GIBI, the teacher facilitates learning by posing questions and providing feedback, while students take greater responsibility for their inquiry process (Berhanu & Sheferaw, 2022; Hastuti et al., 2020).

Empirical evidence supports the GIBI's effectiveness, showing significant improvements in mathematics performance (Arikewuyo et al., 2020; Asanre et al., 2022; Ghazal et al., 2025; Ijeh, 2020; Khasawneh et al., 2023; Ogunjimi & Gbadeyanka, 2023; Wilson, 2020), geometry achievement (Dwianty et al., 2024; Eshetu et al., 2022; Odupe & Opeisa, 2019), and the promotion of mastery goal orientation (Borovay et al., 2019; Mupira & Ramnarain, 2018) when compared with traditional teaching methods. This aligns with the Ethiopian context, where the new mathematics teacher's guide implicitly recommends a guided inquiry method (FDREMOE, 2023).

A prominent model for implementing GIBI is the BSCS 5E Instructional model, grounded in social constructivism, that structures learning into five phases: Engage, Explore, Explain, Elaborate, and Evaluate (Bybee, 2014; Saleem et al., 2021). Studies confirm that the 5E model has a positive impact on students' mathematics achievement and interest (Behera & Bhuyan, 2024; Omotayo & Adeleke, 2017) when compared with their peers who received TTM. Additionally, a comprehensive meta-analysis conducted by Polanin et al. (2024) found that the 5E instructional model was more effective in improving students' STEM education outcomes in grades 9-12 than the 3E and 7E instructional models.

Based on the above findings, the authors of this study found that the 5E instructional model is consistent with the principles of constructivist learning theory and appropriate for implementing GIBI at the classroom level. Thus, all 16 geometric lessons of the experimental groups were designed based on the 5E model. A sample lesson plan of EG1 is found in Appendix B.

Variation theory and teaching geometry

To deepen conceptual engagement within inquiry learning, variation theory provides a powerful framework for designing tasks to direct students' attention to critical aspects of what is to be learned. This review integrates the two complementary theories: Marton's variation learning theory and GU's teaching through variation, also known as "Bianshi" teaching. Marton's theory posits that discernment requires experiencing variation; learning occurs when students become aware of new aspects of an "object of learning" by discerning its "critical aspects" (Kullberg et al., 2017, 2024).

Teachers can facilitate this using specific patterns of variation, such as contrast (varying a critical feature while holding other critical aspects), generalization (holding the critical aspect invariant while varying other features), and fusion (varying several critical aspects simultaneously) (Kullberg et al., 2024; Leung, 2012). Complementarily, GU's teaching through variation offers practical strategies, emphasizing conceptual variation, i.e., presenting a concept in multiple ways (e.g., models, diagrams, nets) to highlight essential features (Gu et al., 2017; Jacques, 2018) and procedural variation that is varying problem-solving contexts and approaches step-by-step, often using scaffolding (i.e., Instructional "Pudian"), to build connections and flexible problem-solving skills (Gu et al., 2017; Pang et al., 2017).

Research demonstrates that variation theory enhances mathematical understanding (Handy, 2021), improves higher-order thinking (Baskoro, 2021), and boosts achievement in specific domains like algebra and geometry (Kassa & Ding, 2019; Jing et al., 2017; Lomibao & Ombay, 2017). Additionally, empirical evidence has shown the importance of using Marton's variation learning theory and GU's teaching through variation together in teaching mathematics, especially in geometry (Kassa & Ding, 2019; Handy, 2021; Pang et al., 2017).

Moreover, GU and his colleagues explained the similarity between Marton's variation theory, which stresses the patterns of what varies and what doesn't, and their conceptual variation (Gu et al., 2017). Additionally, the term scaffolding is associated with social constructivism and involves the teacher providing suitable support for learning, which is similar to instructional "pudian", applied in procedural variation of GU's teaching through variation. This implies that not only the complementary nature of these variation theories, but also that they can be used together with teaching methods, such as GIBI, which is rooted in constructivist learning theory. This complementary relationship is supported by previous studies showing that constructivist pedagogies are significantly enhanced when integrated with variation theory, leading to deeper conceptual understanding (Handy, 2021; Jacques, 2018; Voon et al., 2020).

Goal orientations

A complete understanding of educational interventions also requires examining student motivation, for which achievement goal orientation theory provides a key lens. This theory explains students' purposes for engaging in academic tasks, primarily distinguishing between mastery goals (focus on learning and understanding), performance-approach goals (focus on demonstrating competence), and performance-avoidance goals (focus on avoiding failure) constructs that were understood to help students focus on approaching or moving toward success (Chazan et al., 2022; Elliot & Hulleman, 2017; Niemivirta et al., 2019).

In mathematics, mastery goals are particularly beneficial, as they are positively associated with the use of deep learning strategies, higher achievement (Guo & Hu, 2021; Guo & Leung, 2020), and greater enjoyment, increased behavioral engagement (Putwain et al., 2018). Performance-approach goal-oriented students are motivated by the desire to achieve success, while avoidance-oriented students are motivated by the fear of failure (Yılmaz, 2022). This empirical evidence revealed the benefits of fostering students' achievement goal orientations, especially mastery goals and performance-approach goals, to solve their learning and achievement problems.

Research gap

Despite evidence supporting the efficacy of GIBI and variation theory separately, their integrated application in mathematics education, especially in Sub-Saharan Africa, is scarce, especially in the Ethiopian educational context, where geometry achievement is a persistent challenge (NEAEA, 2017).

Moreover, existing studies have given limited attention to the role of students' motivational beliefs, such as goal orientation, within such innovative teaching interventions (Wang & Xue, 2022). Therefore, this study investigated the effects of GIBI assisted by Variation Theory (GIBI-VT) and on goal orientations and achievement of grade ten students in solid geometry, addressing a critical need for context-specific, evidence-based pedagogical innovations.

Conceptual framework

This study is guided by an integrated conceptual model that combines principles from Guided Inquiry-Based Instruction (GIBI) and Variation Theory (VT), with Achievement Goal Orientation Theory as a lens for understanding motivational outcomes. The model posits that the pedagogical intervention (GIBI-VT or GIBI alone) directly influences students' mathematics achievement. Simultaneously, the intervention, particularly GIBI-VT, is hypothesized to foster a classroom environment that enhances students' achievement goal orientations, which may further support learning outcomes.

Additionally, goal orientations (i.e., mastery goals, performance-approach goals, and performance-avoidance goals) are treated as both outcomes of the intervention and potential predictors of students' achievement in solid geometry. Moreover, students' gender, and their parents' residence were used as potential predictors of students' achievement in solid geometry. This framework allows us to test not only the efficacy of GIBI-VT on achievement but also its motivational mechanisms, addressing a gap in the literature regarding how innovative pedagogies affect both cognitive and affective domains in mathematics education.

METHOD

Research design

In this study, a quasi-experimental with a non-equivalent control group pre-test/post-test design was employed. It involved three study groups: Experimental Group 1 (EG1), who was taught by GIBI Assisted by variation theory (GIBI-VT); Experimental Group 2 (EG2), who were instructed by GIBI alone; and the Control Group (CG), who were taught by Traditional Teaching Methods (TTM). All groups underwent pre-testing and post-testing as shown in Table 1 below.

Table 1

Research design layout

Study Groups		Intervention	
EG1	Pre-Test	GIBI-VT	Post-Test
EG2	Pre-Test	GIBI alone	Post-Test
CG	Pre-Test	TTM	Post-Test

Sampling techniques

A multi-stage sampling method was used in this study. First, of the four government secondary schools in Debre Tabor City, Ethiopia, three schools were purposively selected based on infrastructure and class size. Then, from these schools three mathematics teachers, one from each school, were chosen purposively based on gender, teaching experience, Education level, and willingness to participate. Finally, three classes from the teachers' intact classes were randomly selected and assigned to the study groups (i.e., EG1, EG2, and CG). These classes contained a total of 102 students.

Instrumentation

Mathematics achievement test

The Mathematics Achievement (MA) test was developed by the author of this study based on the new FDREMOE (2023) grade ten mathematics curricula to measure students' achievement in solid geometry before and after the intervention. The original version of the MA test consisted of 42 items (35 multiple-choice, 7 short answers), which was developed by preparing a table of specifications, and the revised Bloom taxonomy to ensure instructional content validity. Its face validity was ensured by advisors, two mathematics teachers, and one mathematics lecturer of Begemider College of Teacher Education (BCTE).

After corrections were made based on their comments, it was piloted with a sample of 38 grade 12 students (21 female and 17 male) of Debre Tabor Secondary School (DTSS) who were assumed they have completely covered grade ten mathematics contents to calculate its item difficulty, item discrimination index, and reliability.

Based on the item discrimination, item difficulty, and effectiveness of alternatives results, seven items were removed, and then the final MA test had 36 multiple-choice items with four response options (A, B, C, and D) as shown in Appendix A. The final test items' difficulty index (P) values range from 0.11 to 0.63, with an average value of 0.35. Besides, its reliability coefficient, calculated using Kuder–Richardson formulas (K-R20), was 0.72, indicating an internal consistency of the test items. It was implemented before and after the intervention, and scored out of 36 marks. The mathematic achievement questions used as pre-test in found in Appendix A.

Goal orientation questionnaire

The questionnaire, which was adapted from instruments used in previous studies, contained nine demographic items and 11 items with a five-point Likert scale (1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree, and 5 = strongly agree) that addressed students' mastery, performance-approach, and performance-avoidance goals. Parents' residence was recorded as urban or rural based on students' self-reports at pre-test. Face and content validity were confirmed by advisors and educational psychologists of BCTE. The Amharic version was piloted with 39 grade ten students of DTSS who weren't part of this study, and after removing three poorly performing items, the final 8-item scale demonstrated good internal consistency (Cronbach's $\alpha = 0.715$).

Implementation procedures

Before the intervention, a supplementary manual was prepared based on GIBI and variation theory using the 5E instructional model. The manual contained 16 lesson plans for 16 different solid geometry activities, each lasting 45 minutes.

Subsequently, a week-long training session from May 4, 2024, to May 10, 2024, was organized for two mathematics teachers who taught the experimental groups in the corresponding author's office at BCTE. The training aimed to fill teachers' knowledge gaps on the principles of GIBI, variation theory, and the 5E instructional model; to show how they can implement the teaching method in their classrooms; to explain how they can scaffold their students during implementation; and to validate the manual.

Following this, the teacher who taught EG1 received an additional one-day training on May 11, 2024, about how to use geometric activities designed based on variation theory. At the end of the training, the teachers who taught experimental groups get all the necessary teaching and learning materials.

A month before the intervention, a pre-test was administered to collect data on students' baseline in goal orientations and achievement in solid geometry. In a heterogeneous structure, the teachers formed six groups with four to five students for the experimental groups, while the CG underwent TTM in their natural setting. The intervention took four weeks with four periods per week. After the intervention, with some reshuffling of pre-test items, the post-test was administered.

Data analysis

Quantitative data were analyzed using SPSS version 24 software. Preliminary analyses checked assumptions for parametric tests. To address the three research questions specifically: For RQ1 (i.e., achievement difference), a one-way Analysis of Covariance (ANCOVA) compared post-test scores with pre-test as covariate. Additionally, for RQ2 (i.e., achievement goal orientation sub-constructs differences), a one-way Multivariate Analysis of Variance (MANOVA) examined the three achievement goal orientation sub-constructs simultaneously. Finally, for RQ3 (predictors of achievement in solid geometry), a stepwise multiple regression analysis was conducted with teaching methods, goal orientations, students' gender, and their parents' residence as potential predictors of achievement in solid geometry.

RESULTS

Mathematics achievement differences

RQ1. Is there a significant difference in students' mathematics achievement post-test scores between the experimental and control groups after controlling for their pre-test scores? To answer this research question, a one-way ANCOVA was conducted with the teaching method as the independent variable, mathematics achievement post-test scores (MAPostTS) as the dependent variable, and MAPreTS as the covariate (see **Table C1**) in the Appendix C. Prior to computing, all assumptions of ANCOVA were checked (see **Table D** (**1-3**) and **Figure D1**) in the Appendix D. After adjusting for pre-test scores, the results in **Table 2** revealed a statistically significant difference among the three groups on MAPostTS ($F(2, 95) = 19.88$, $p = .000$) with a large effect size, partial $\eta^2 = .295$, indicating that 29.5% of the variance in post-test achievement was accounted for by the intervention.

Post-hoc pairwise comparisons using the Bonferroni correction in **Table 3** showed that both EG1 ($M = 12.27$, $SD = 3.41$) and EG2 ($M = 10.76$, $SD = 2.33$) scored significantly higher than the CG ($M = 7.13$, $SD = 1.81$) ($p = .000$ for both comparisons). Although EG1 had a higher mean score than EG2, this difference wasn't statistically significant ($p = 1.000$).

Goal orientation sub-constructs differences

RQ2. Is there a significant difference in students' goal orientation sub-constructs post-test scores between the experimental and control groups? In this research question students' goal orientations sub-constructs contained their Mastery Goals Post-Test Scores (MaGPostTS), Performance-Approach Post-Test Scores (PApPostTS), and Performance- Avoidance Post-Test Scores (PAvPostTS). It was addressed using a one-way between-group MANOVA to examine the effect of the teaching methods on these goal orientation sub-constructs. Before executing MANOVA, we have checked the tenability of all its assumptions. The results are found in Appendix E. The multivariate test in Table 4 indicated a significant overall effect (Pillai's Trace = .128, F (6, 194) = 2.24, p = .041, partial η^2 = .064) on students' GOPostTS in solid geometry.

Follow-up univariate tests (Tests of Between-Subjects Effects), Table 5 revealed that this significant effect was specific to Mastery Goal Orientation Post-Test Scores (MaGPostTS), F (2, 99) = 5.97, p = .004, η^2 = .108, while no significant differences were found for performance-approach and performance-avoidance goals post-test scores (p > .05).Post-hoc comparisons using the Games-Howell in Table 6 indicated that EG1 scored significantly higher on mastery goals than EG2 (p = .036) and the CG (p = .001). There was no significant difference in mastery goals between EG2 and the CG (p = .570).

Predictors of mathematics achievement

RQ3. What are the contributions of students' gender, their parents' residence (Urban vs. Rural), teaching methods (GIBI-VT vs. TTM, and GIBI alone vs. TTM), and goal orientation sub-constructs on their achievement in solid geometry? To answer this research question, a stepwise regression analysis was executed by ensuring the non-violation of its assumptions (see Tables F (1-3) and Figure F (1-2)) in the Appendix F. The analysis aimed to explain students' MAPostTS based on their gender, parents' residence, the teaching methods, and their GO sub-constructs post-test scores.

The final model in Table 7 revealed that students' gender, performance - approach post-test scores (PApPostTS), GIBI-VT, and GIBI alone explained 47.0% of the variance in their MAPostTS (R^2 = .470, Adjusted R^2 = .447). The model was statistically significant, F (4, 94) = 20.83, p = .000.

Table 8 revealed four significant predictors of mathematics achievement. The most substantial predictors were the teaching methods. Being in the EG1 that received GIBI-VT was a strong positive predictor (β = .683, p = .000), as was being in the EG2 that received GIBI alone (β = .538, p = .000), both compared to the TTM of CG. Furthermore, students' PApPostTS was a significant, though smaller, positive predictor of their achievement (β = .185, p = .018) in solid geometry.

Finally, gender was also a significant predictor (β = .159, p = .038), with male students scoring higher on average than female students. However, students' mastery goal orientation, performance-avoidance goal orientation, and their parents' residence weren't significant predictors in the final model.

Table 2

ANCOVA result of MAPostTS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	586.729a	3	195.576	38.143	.000	.546
Intercept	936.445	1	936.445	182.633	.000	.658
MAPreTS	147.069	1	147.069	28.683	.000	.232
Groups	203.908	2	101.954	19.884	.000	.295
Error	487.109	95	5.127			
Total	11054.000	99				
Corrected Total	1073.838	98				

Note. a. R Squared = .546 (Adjusted R Squared = .532)

Table 3

Pairwise comparisons for MAPostTS

					95% Confidence Interval for Difference[b]	
(I) Study Groups	(J) Study Groups	Mean Difference(I-J)	Std. Error	Sig.b	Lower Bound	Upper Bound
EG1	EG2	.557	.584	1.000	-.866	1.981
	CG	3.599*	.644	.000	2.030	5.167
EG2	CG	3.041*	.558	.000	1.682	4.400

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Bonferroni.

Table 4

Multivariate tests for GOPostTS

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Groups	Pillai's Trace	.128	2.238	6.000	196.000	.041	.064
	Wilks' Lambda	.873	2.269a	6.000	194.000	.039	.066
	Hotelling's Trace	.144	2.299	6.000	192.000	.036	.067
	Roy's Largest Root	.132	4.308b	3.000	98.000	.007	.117

a. Exact statistic

b. The statistic is an upper bound on F that yields a lower bound on the significance level.

Table 5

Tests of between-subjects effects for GO sub–constructs

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	MaGPostTS	69.039	2	34.520	5.971	.004	.108
	PApPostTS	19.668	2	9.834	1.907	.154	.037
	PAvPostTS	4.987	2	2.494	.707	.496	.014
Intercept	MaGPostTS	11723.716	1	11723.716	2027.923	.000	.953
	PApPostTS	12044.730	1	12044.730	2335.633	.000	.959
	PAvPostTS	4547.995	1	4547.995	1288.912	.000	.929
Groups	MaGPostTS	69.039	2	34.520	5.971	.004	.108
	PApPostTS	19.668	2	9.834	1.907	.154	.037
	PAvPostTS	4.987	2	2.494	.707	.496	.014
Error	MaGPostTS	572.333	99	5.781			
	PApPostTS	510.538	99	5.157			
	PAvPostTS	349.327	99	3.529			
Total	MaGPostTS	12418.000	102				
	PApPostTS	12675.000	102				
	PAvPostTS	4968.000	102				
Corrected Total	MaGPostTS	641.373	101				
	PApPostTS	530.206	101				
	PAvPostTS	354.314	101				

Table 6

Games-Howell multiple comparisons test

Dependent Variable		(I) Study Groups	(J) Study Groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
MaGPostTS	Games-Howell	EG1	EG2	1.45*	.571	.036	.08	2.82
			CG	2.03*	.520	.001	.78	3.28
		EG2	CG	.58	.604	.603	-.86	2.03

*. The mean difference is significant at the .05 level.

Table 7

Regression model summary for MAPostTS

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change	Durbin-Watson
1	.446a	.199	.190	2.978	.199	24.048	1	97	.000	
2	.640b	.409	.397	2.570	.211	34.261	1	96	.000	
3	.667c	.445	.427	2.505	.036	6.084	1	95	.015	
4	.685d	.470	.447	2.461	.025	4.415	1	94	.038	1.933

a. Predictors: (Constant), GIBI-VT vs. TTM

b. Predictors: (Constant), GIBI -VT vs. TTM, GIBI alone vs. TTM

c. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM, PApPostTS

d. Predictors: (Constant), GIBI -VT vs. TTM, GIBI alone vs. TTM, PApPostTS, Student's Gender

Table 8

Regression coefficients of predictive variables

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	9.072	.359		25.302	.000
	GIBI-VT vs. TTM	3.194	.651	.446	4.904	.000
2	(Constant)	7.125	.454		15.682	.000
	GIBI-VT vs. TTM	5.142	.653	.717	7.872	.000
	GIBI alone vs. TTM	3.632	.620	.533	5.853	.000
3	(Constant)	4.246	1.249		3.400	.001
	GIBI-VT vs. TTM	4.839	.648	.675	7.465	.000
	GIBI alone vs. TTM	3.542	.606	.520	5.848	.000
	PApPostTS	.277	.112	.192	2.467	.015
4	(Constant)	3.939	1.235		3.189	.002
	GIBI-VT vs. TTM	4.897	.638	.683	7.682	.000
	GIBI alone vs. TTM	3.663	.598	.538	6.126	.000
	PApPostTS	.266	.110	.185	2.410	.018
	Student's Gender	1.117	.532	.159	2.101	.038

Note: Gender coded as 1 = Male, 0 = Female

DISCUSSION

The findings of this study strongly support the effectiveness of innovative pedagogies in enhancing learning outcomes in solid geometry. The significant improvement in mathematics achievement for both experimental groups (EG1 and EG2) over the CG confirms that both GIBI based on variation theory and GIBI alone were effective in enhancing students' achievement in solid geometry. This finding aligns with a substantial body of research advocating for inquiry-based learning (Arikewuyo et al., 2020; Asanre et al., 2022; Dwianty et al., 2024; Ijeh, 2020; Khasawneh et al., 2023; Odupe & Opeisa, 2019; Ogunjimi & Gbadeyanka, 2023; Ogbaga et al., 2024; Çetinavcı, 2021; Lee et al., 2023). However, this finding contrasts with Aguguam & Okoro (2020), Richter et al. (2022), Sizemore (2020), and Owolade et al. (2022), who found that TTM and open inquiry were more effective than GIBI.

The fact that EG1 (GIBI-VT) outperformed EG2 (GIBI alone), albeit not at a statistically significant level, suggests a meaningful trend. This can be attributed to the structured variation, which likely helped students discern critical features of geometric concepts more effectively, thereby deepening their conceptual understanding, a finding consistent with the principles of variation theory (Gu et al., 2017; Kullberg et al., 2017), and previous studies (Aydın-Güç, 2021; Baskoro, 2021; Jing et al., 2017; Mendoza & Lapinid, 2024; Yujia, 2020; Halpern et al., 2025; Bakir & Banikhalaf, 2025).

A pivotal finding is the significant positive effect of GIBI-VT on students' mastery goal orientation. EG1 reported significantly higher mastery goals than both other groups, indicating that this integrated approach not only taught content but also fostered a learning environment where students were more focused on understanding and developing competence, which is consistent with the findings of Mupira & Ramnarain (2018).

The regression analysis further clarifies the dynamics of achievement. The strongest predictors were the teaching methods: GIBI-VT and GIBI alone when compared with TTM, confirming their direct impact. The significant role of performance-approach goals, rather than mastery goals, in predicting achievement scores is intriguing. It may be that the desire to demonstrate competence is a more immediate driver of test performance. This finding is in line with the findings of Deraja et al. (2023) and Guo & Leung (2020), Dogutas (2025), and contrasts with Guo & Hu (2021), who found that students' performance-approach orientation had a non-significant relationship with mathematics achievement.

Additionally, students' mastery goal wasn't a significant predictor of their mathematics achievement, which contradicts Deraja et al. (2023), Gidado et al. (2025), Guo & Hu (2021), Wang & Xue (2022), and Zhong et al. (2023), who found that students' mastery goal is a strong positive predictor of their mathematics achievement. Finally, the finding that students' gender was a significant predictor, though with a small effect, aligns with studies suggesting boys often report higher confidence in spatial tasks (geometry) (Grewe, 2025). Future GIBI-VT designs could incorporate gender-sensitive scaffolding to support girls' engagement.

CONCLUSION

This study provides compelling empirical evidence that GIBI-VT is a powerful, scalable pedagogical strategy for enhancing both achievement and motivation in mathematics, specifically in the challenging domain of solid geometry. While situated in the Ethiopian context, the findings offer significant insights and actionable strategies for educators, curriculum developers, and policymakers worldwide who are grappling with low achievement and declining motivation in STEM education.

Globally, mathematics education faces the dual challenge of moving beyond rote memorization and fostering deep conceptual understanding alongside intrinsic motivation. This research demonstrates that the integration of two robust frameworks, inquiry-based learning and variation theory, creates a synergistic effect that addresses both challenges. Students taught with GIBI-VT not only achieved significantly higher scores than their peers in traditional classrooms but also developed stronger mastery goal orientations, a motivational profile linked to perseverance, deep learning, and long-term academic resilience. This outcome is particularly critical for global education systems that aim to cultivate learners who are adaptable, critical thinkers, rather than just proficient test-takers.

The study's contribution extends beyond proving efficacy. It offers a replicable model for pedagogical design, the 5E instructional model structured by patterns of variation (contrast, generalization) that can be adapted across diverse cultural and educational settings. The finding that GIBI-VT specifically fosters mastery goals suggests that it can help shift classroom cultures worldwide from a focus on performance and competition to one on understanding and growth.

Furthermore, the research highlights that innovative pedagogy is a key lever for equity. In contexts like Ethiopia, and by extension, in underserved educational regions everywhere, the strategic shift from teacher-centered instruction to structured, concept-focused inquiry can be a powerful tool for closing achievement gaps.

The study also responsibly identifies areas for further global investigation, such as the nuanced role of gender and the specific mechanisms by which variation theory enhances conceptual discernment.

In conclusion, this work transcends its local context to provide an evidence-based blueprint for transforming geometry instruction internationally. It argues convincingly that the future of effective mathematics education lies in pedagogies that are simultaneously structured (through theoretical frameworks like variation theory) and student-centered (through guided inquiry). As educational systems worldwide strive to meet the demands of the 21st century, the GIBI-VT approach stands out as a practical, theoretically sound, and highly effective strategy for developing both the skilled and motivated learners our global society needs.

RECOMMENDATIONS

Based on the findings, the following recommendations are proposed.

- The Ethiopian Ministry of Education should consider integrating the principles of variation theory and guided inquiry into the national mathematics curriculum and teacher guides. Professional development programs should be designed to train in-service teachers on how to design and implement 5E-based GIBI lessons that incorporate different patterns of variation.
- Mathematics teachers should encourage the adoption of guided inquiry in their geometry lessons. They should deliberately use contrast and generalization patterns of variation when introducing concepts (e.g., presenting prisms alongside pyramids and cones) to help students discern critical features.
- Since GIBI-VT increased mastery goal orientation, teacher training should include variation theory patterns (contrast, generalization) to foster intrinsic motivation alongside conceptual learning.
- Future studies should conduct a longitudinal study to see the long-term impact of GIBI-VT on advanced mathematics performance. Researchers should also qualitatively explore the reasons behind the gender difference found in this study.

Acknowledgement

The authors express their gratitude to the principals of the three secondary schools in Debre Tabor City, Amhara Region**,** Ethiopia, for their invaluable support in doing this study.

Funding

No funding source is reported for this study.

Ethical statement

An ethical approval sheet was obtained from Hawassa University. Additionally, written informed consents were obtained from teachers, and verbal assent was obtained from students after explaining the study's purpose, voluntary nature, and their right to withdraw at any time without consequence.

Competing interests

The authors declared no competing interest.

Author contributions

Yeshanew: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Writing - original draft, Writing - review & editing; Belachew: Conceptualization, Methodology, Supervision, Validation, Writing - review & editing; Gezahegn: Methodology, Supervision, Validation Writing - review & editing; Tesfa: Supervision, Validation Writing - review & editing.

Data availability

N/A

AI disclosure

In this study, we utilized "Grammarly" and "DeepSeek" for writing quality and language improvement.

Biographical sketch

Abebaw Yeshanew is a Doctor of Education (D.Ed.) student in the Department of Mathematics at Hawassa University, Ethiopia. He received the Masters of Education (M.Ed.) in mathematics from Bahir Dar University in the year 2008. He has more than 12 years of experience in teaching at different College of Teachers Educationin Ethiopia. His research interests include Mathematics education, Psychology in mathematics, Technology and Mathematics learning. He can be reached with y.abebaw2012@gmail.com.

Tesfu Belachew, Ph.D., is an Associate Professor and at Madda Walabu University, Ethiopia. With a PhD in Mathematics Education and nearly 30 years of experience, he has served as a lecturer, college director. His research focuses on students' beliefs in mathematics and instructional methods, with multiple peer-reviewed publications. An active academic examiner and advisor, he also contributes to curriculum development, and teacher training. He is dedicated to enhancing mathematics education through research and community-focused initiatives. He can be reached with tesfubelachew3@gmail.com.

Habtamu Gezahegn, Ph.D., is an Associate Professor in the Department of Educational Planning and Management at Hawassa University, Ethiopia. His expertise lies Educational Management, Educational Policy, and Educational Leadership. He has published numerous articles on his expert fields. He can be reached with ghabtamu59@gmail.com.

Tadele Tesfa is an Assistant Professor in the Department of Mathematics at Hawassa University, Ethiopia. He holds a PhD degree in Mathematical Modeling and has more than 15 years of experience in teaching and research at the university level. His research interests include mathematical modeling and mathematics education, with a focus on applying mathematical methods to real-world problems and improving the teaching and learning of mathematics. He can be reached with tadelet@hu.edu.et.

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APPENDICES

Appendix A. Mathematics Achievement Pre-Test

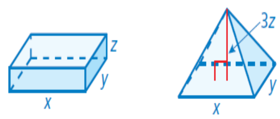
Section____________ **No** ______________**School Name** ___________**Time allotted:** 80'

Instruction: ***Please circle the letter of your choice from the given alternatives.***

1. Which of the following is not a prism?
   A) Cube B) Rectangular solid C) Parallelepiped D) Tetrahedron
2. What plane figures are the lateral faces of a right regular pyramid?
   A) Scalene Triangles B) Trapeziums C) Isosceles Triangles D) Rectangles
3. The lateral face of frustum of a right cone is ________
   A) Sector of annulus B) Trapezium C) Circle D) Isosceles triangles
4. Which of the following object has a spherical shape?
   A) Tea Cup B) Matchbox C) Moon D) Happy birthday Cape
5. The lateral faces of frustum of a regular pyramid are ___________
   A) Rectangles B) Isosceles Trapeziums C) Trapeziums D) Parallelogram

Instruction: ***Items numbered 6 to 17 have two options. So, make sure in selecting from the two options.***

6. The radius of a spherical balloon increases from 7cm to 14 cm when air is pumped into it. The *ratio* of the surface area of the original balloon to the inflated one is _______
   A) 1:2 B) 2:1 C) 1:4 D) None of these
   The reason for your answer above is:
   Let r be the radius of a spherical balloon. Then the surface area (***SA***) of the balloon becomes $4\pi r^2$. So,
   A) $\frac{SA\ of\ the\ original\ balloon}{SA\ of\ the\ inflated\ balloon} = \frac{4\pi(7)^2}{4\pi(14)^2} = \frac{14}{28} = \frac{1}{2}$ (i.e., 1: 2)
   B) $\frac{SA\ of\ the\ original\ balloon}{SA\ of\ the\ inflated\ balloon} = \frac{4\pi(7)^2}{4\pi(14)^2} = (\frac{1}{2})^2 = \frac{1}{4}$ (i.e., 1: 4)
   C) $\frac{SA\ of\ the\ inflated\ balloon}{SA\ of\ the\ original\ balloon} = \frac{4\pi(14)^2}{4\pi(7)^2} = \frac{28}{14} = 2$ (i.e., 2: 1)
   D) None of these
7. If the diagonal of a cube is d cm, what is the *volume* of the cube?
   A) $\frac{d^3}{\sqrt{3}}$ cm³ B) $\frac{d^3}{\sqrt{2}}$ cm³ C) $\frac{d^3}{3\sqrt{3}}$ cm³ D) $\frac{d^3}{2\sqrt{2}}$ cm³
   The reason for your answer above is:
   If the length of one side of a cube is a cm, then its volume (***V***) becomes a^3 . Thus,
   A) $a = \frac{d}{\sqrt{3}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{3}})^3 = \frac{d^3}{3\sqrt{3}}$ B) $a = \frac{d}{\sqrt{2}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{2}})^3 = \frac{d^3}{2\sqrt{2}}$
   C) $a = \frac{d}{\sqrt{3}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{3}})^3 = \frac{d^3}{\sqrt{3}}$ D) $a = \frac{d}{\sqrt{2}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{2}})^3 = \frac{d^3}{\sqrt{2}}$
8. If the total surface area of a cube is $x\ cm^2$ and its volume is $\frac{x\sqrt{3}}{3}cm^3$, find the *Main diagonal* of the cube.
   A) $6\ cm$ B) $4\ cm$ C) $2\sqrt{6}\ cm$ D) $3\ cm$
   The reason for your answer above is:
   Let a and c be the length of one edge and side diagonal of the cube respectively. Then
   A) $4a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{4}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{4}) = \frac{x\sqrt{3}}{3} \Rightarrow a = \frac{4\sqrt{3}}{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}\left(\frac{4\sqrt{3}}{3}\right) = 4$
   B) $3a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{3}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{3}) = \frac{x\sqrt{3}}{3} \Rightarrow a = \sqrt{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}(\sqrt{3}) = 3$
   C) $6a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{6}$, but $c = a\sqrt{2}$ So, $a^3 = a(a^2) = a(\frac{x}{6}) = \frac{x\sqrt{3}}{3} \Rightarrow a = 2\sqrt{3} \Rightarrow d = a\sqrt{2} = \sqrt{2}(2\sqrt{3}) = 2\sqrt{6}$
   D) $6a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{6}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{6}) = \frac{x\sqrt{3}}{3} \Rightarrow a = 2\sqrt{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}(2\sqrt{3}) = 6$
9. Which of the following is **true** about the *volume* of two solids figures shown below? Why?
   A) The volume of prism is greater than the volume of pyramid
   B) They have the same volume
   C) The volume of prism is less than the volume of pyramid
   D) I can't compare their volume



   The reason for your answer above is:
   A) Although the height of the pyramid is three times higher than the height of the prism, the volume of prism is greater than the volume of pyramid
   B) The volume of the prism is base area times height (i.e. XYZ) and the volume of the pyramid is one-third times base area times height (i.e. XYZ). So, they have the same volume.
   C) I can't compare their volume because their dimensions are only variables.

D) The volume of the prism is one-third times base area times height (i.e.$\frac{1}{3}$ XYZ) and the volume of the pyramid is base area times height (i.e. 3XYZ). So, the volume of prism is less than the volume of pyramid

10. The sum of the bases area of a cylinder is **equal** to its lateral face area. If the altitude of the cylinder is 2 cm, then its *volume* is _______

A) $\frac{32}{3}\pi\ cm^3$ B) $4\pi\ cm^3$ C) $16\pi\ cm^3$ D) $8\pi cm^3$

The reason for your answer above is:

Let $\boldsymbol{r}$ and $\boldsymbol{h}$ be the radius and height of the cylinder respectively. Then,

A) $2\pi r^2 = 2\pi rh$ & $h = 2 \Rightarrow r = 2$. So, its volume (V) = $\frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(2) = 4\pi$

B) $\pi r^2 = 2\pi rh$ & $h = 2 \Rightarrow r = 4$. So, its volume (V) = $\frac{1}{3}\pi r^2 h = \pi(4)^2(2) = \frac{32}{3}\pi$

C) $2\pi r^2 = 2\pi rh$ & $h = 2 \Rightarrow r = 2$. So, its volume (V) = $\pi r^2 h = \pi(2)^2(2) = 8\pi$

D) $\pi r^2 = 2\pi rh$ & $h = 2 \Rightarrow r = 4$. So, its volume (V) = $\frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(4)^2(2) = 16\pi$

11. How many **edges** does an oblique pentagonal pyramid have?

A) 6 B) 10 C) 5 D) 8

The reason for your answer above is:

A) Since the base of the pyramid is pentagon (i.e. five sided polygon) and its faces are triangles (i.e. three sided polygon), the number of edges of the pyramid becomes 8.

B) Since the base of the pyramid is pentagon (i.e. five sided polygon) and its faces are triangles that meet at one point, the number of edges of the pyramid becomes 6.

C) Since the base of the pyramid is pentagon (i.e. five sided polygon), the number of edges of the pyramid becomes 5.

D) Since the base of the pyramid is pentagon (i.e. five sided polygon) and its faces are triangles that have five common sides, the number of edges of the pyramid becomes 10.

12. A triangular pyramid and a triangular prism have the **same** base and height. How many times the volume of the *prism* is greater than the volume of the *pyramid*?

A) 2 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 3

The reason for your answer above is:

Let $\boldsymbol{V_1}$ be the volume of the prism and $\boldsymbol{V_2}$ be the volume of the pyramid. Then,

A) $V_1 =$ (base area)(height) & $V_2 = \frac{1}{3}$ (base area)(height) $\Rightarrow V_1 = 3V_2$

B) $V_1 =$ (base area)(height) & $V_2 = \frac{1}{2}$ (base area)(height) $\Rightarrow V_1 = 2V_2$

C) $V_2 =$ (base area)(height) & $V_1 = \frac{1}{3}$ (base area)(height) $\Rightarrow V_1 = \frac{1}{3}V_2$

D) $V_2 =$ (base area)(height) & $V_1 = \frac{1}{2}$ (base area)(height) $\Rightarrow V_1 = \frac{1}{2}V_2$

13. If each edge of a regular tetrahedron is 12 cm, then its lateral surface area is ________

A) $216\sqrt{3}\ cm^2$ B) $108\sqrt{3}cm^2$ C) $144\sqrt{3}cm^2$ D) $288\sqrt{3}cm^2$

The reason for your answer above is:

All faces of a tetrahedron are equilateral triangles whose edge is *s* =12 cm long. So,

A) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{4} = \frac{(12\ cm)^2\sqrt{3}}{4} = \frac{144\sqrt{3}cm^2}{4} = 36\sqrt{3}cm^2 \Rightarrow$ its lateral surface area becomes $4(36\sqrt{3}cm^2) = 144\sqrt{3}cm^2$

B) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{2} = \frac{(12\ cm)^2\sqrt{3}}{2} = \frac{144\sqrt{3}cm^2}{2} = 72\sqrt{3}cm^2 \Rightarrow$ its lateral surface area becomes $3(72\sqrt{3}cm^2) = 216\sqrt{3}cm^2$

C) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{4} = \frac{(12\ cm)^2\sqrt{3}}{4} = \frac{144\sqrt{3}cm^2}{4} = 36\sqrt{3}cm^2 \Rightarrow$ its lateral surface area becomes $3(36\sqrt{3}cm^2) = 108\sqrt{3}cm^2$

D) D) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{2} = \frac{(12\ cm)^2\sqrt{3}}{2} = \frac{144\sqrt{3}cm^2}{2} = 72\sqrt{3}cm^2 \Rightarrow$ its lateral surface area becomes $4(72\sqrt{3}cm^2) = 288\sqrt{3}cm^2$

14. The *diameter* of the Moon is approximately **one-fourth** of the *diameter* of the Earth. What *fraction* of the volume of the Earth is the volume of the Moon?

A) $\frac{1}{64}$ B) $\frac{1}{512}$ C) $\frac{1}{16}$ D) $\frac{1}{12}$

The reason for your answer above is:

Let r = radius of Moon; R = radius of Earth; V_{Moon} is the volume of the Moon and V_{Earth} is the volume of the Earth. Then,

A) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{1}{4}R)^3 = \frac{1}{12}[\frac{4}{3}\pi R^3] = \frac{1}{12}V_{Earth}$

B) $r = \frac{1}{8}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(\frac{1}{8}R)^3 = \frac{1}{512}[\frac{4}{3}\pi R^3] = \frac{1}{512}V_{Earth}$

C) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{1}{4}R)^3 = \frac{1}{64}[\frac{4}{3}\pi R^3] = \frac{1}{64}V_{Earth}$

D) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(\frac{1}{4}R)^2 = \frac{1}{16}[\frac{4}{3}\pi R^2] = \frac{1}{16}V_{Earth}$

15. What is the volume of a right cone with base diameter 21 cm and height 4 cm?

A) 441π cm3 B) 294π cm3 C) 220.5π cm3 D) 147π cm3

The reason for your answer above is:

Given: diameter (***d***) = 21 m & height (***h***) = 4 m. Let ***V*** be volume of the right cone. Then,

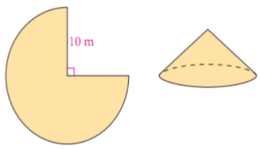
A) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{21}{2}m\right)^2(4m) = \frac{1}{3}(21)^2\pi\, m^3 = \frac{1}{3}(441\pi\, m^3) = 147\pi\, m^3$

B) $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi\left(\frac{21}{2}m\right)^2(4m) = \frac{1}{2}(21)^2\pi\, m^3 = \frac{1}{2}(441\pi\, m^3) = 220.5\pi\, m^3$

C) $V = \frac{1}{4}\pi r^2 h = \frac{1}{4}\pi(21m)^2(4m) = (21)^2\pi\, m^3 = 147\pi\, m^3$

D) $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi\left(\frac{21}{2}m\right)^2(4m) = 21^2\pi\, m^3 = 441\pi\, m^3$

16. A cone is formed from sector of a disk that has radius 10 cm as you see the side figure. What is the lateral surface area of the cone?



A) $\frac{25}{4}\pi\, cm^2$ B) $\frac{75}{4}\pi\, cm^2$ C) $75\pi\, cm^2$ D) None of these

The reason for your answer above is:

The lateral face area (*LA*) of the cone is equal to the area of the sector with radius(***r***) and central angle (***θ***). Thus,

A) $LA = \frac{\pi r^2\theta}{360^0} = \frac{\pi(5\,cm)^2 270^0}{360^0} = \frac{75}{4}\pi\, cm^2$

B) $LA = \frac{\pi r^2\theta}{360^0} = \frac{\pi(10\,cm)^2 270^0}{360^0} = 75\pi\, cm^2$

C) $LA = \frac{\pi r^2\theta}{360^0} = \frac{\pi(5\,cm)^2 90^0}{360^0} = \frac{25}{4}\pi\, cm^2$

D) None of these

17. What is the surface area of the sphere if its volume is $\frac{32\pi}{3}cm^3$?

A) $32\pi\, cm^2$ B) $72\pi\, cm^2$ C) $12\pi\, cm^2$ D) $16\pi\, cm^2$

The reason for your answer above is:

Let ***r***, ***SA*** and ***V*** be the radius, surface area and volume of the sphere respectively. Then,

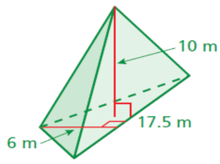
A) $V = \frac{4}{3}\pi r^3 = \frac{32\pi}{3}cm^3 \Rightarrow r^3 = 8\,cm^3 \Rightarrow r = 2\,cm$. So, $SA = 4\pi r^2 = 4\pi(2\,cm)^2 = 16\pi\, cm^2$

B) $V = \frac{2}{3}\pi r^2 = \frac{32\pi}{3}cm^3 \Rightarrow r^2 = 12\,cm^2 \Rightarrow r = 4\,cm$. So, $SA = 2\pi r^2 = 2\pi(4\,cm)^2 = 32\pi\, cm^2$

C) $V = \frac{1}{3}\pi r^3 = \frac{32\pi}{3}cm^3 \Rightarrow r^3 = 32\,cm^3 \Rightarrow r = 6\,cm$. So, $SA = \frac{1}{3}\pi r^2 = \frac{1}{3}\pi(6\,cm)^2 = 12\pi\, cm^2$

D) $V = \frac{1}{3}\pi r^3 = \frac{32\pi}{3}cm^3 \Rightarrow r^3 = 32\,cm^3 \Rightarrow r = 6\,cm$. So, $SA = 2\pi r^2 = 2\pi(6\,cm)^2 = 72\pi\, cm^2$

18. The volume of the right triangular pyramid shown to the side is ___________



A) $175\, m^3$ B) $355\, m^3$ C) $725\, m^3$ D) None of these

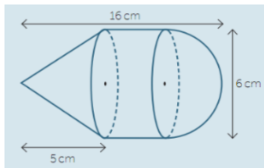
19. What is the total surface area of the rectangular prism with the dimensions 3cm, 4cm and 5cm?

A) 94 cm2 B) 82 cm2 C) 70 cm2 D) 112 cm2

20. The volumes of a cylinder and a sphere with equal radii ***r*** are equal. The altitude of the cylinder in terms of r is ________

A) $4r$ B) $2r$ C) $\frac{4r}{3}$ D) $\frac{r}{3}$

21. Find the volume of the following composed figure shown to the side.



A) $123\pi\, cm^3$ B) $153\pi\, cm^3$ C) $150\pi\, cm^3$ D) $114\pi\, cm^3$

22. If the total surface area of a regular square pyramid is 144 cm2 and length of one side of its base is 8 cm, then the volume of the pyramid is ________

A) 8 cm^3 B) 64 cm^3 C) 80 cm^3 D) 106.6 cm^3

23. A cone has a volume of 600πcm3 and a height of 50 cm. What is the radius of the cone?

A) 3.5 cm B) 6.0 cm C) 10.6 cm D) 36.0 cm

24. A conical tent is 10 m high and the radius of its base is 24 m. The slant height of the tent is _____

A) 26m B) 28m C) 25m D) 27m

25. The lateral surface area of a cone is $308\ cm^2$ and its slant height is 14 cm. The radius of its base is_____

A) $\frac{14}{\pi}$ cm B) $\frac{20}{\pi}$ cm C) $\frac{21}{\pi}$ cm D) $\frac{22}{\pi}$ cm

26. The total surface area of hemisphere of radius 10 cm is________ [Use $\pi = 3.14$]

A) 842 cm^2 B) 940 cm^2 C) 942 cm^2 D) 840 cm^2

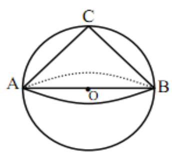
27. The surface area of a sphere with diameter 14 cm is______

A) 784π cm^2 B) 588π cm^2 C) 392π cm^2 D) 299π cm^2

28. If radius of a sphere is $\frac{2a}{3}$ cm, then its volume is _______

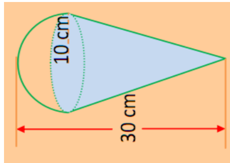
A) $\frac{32}{81}\pi a^3 cm^3$ B) $\frac{23}{4}\pi a^3 cm^3$ C) $\frac{32}{3}\pi a^3 cm^3$ D) $\frac{34}{3}\pi a^3 cm^3$

29. In the given figure, what is the ratio of the volume of the sphere to the volume of the cone?

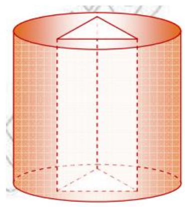


A) 2 B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) 4

30. The base area of a regular square pyramid is 100 cm2 and the sum of the area of its lateral faces is 260 cm2. What is the altitude of the pyramid?
A) 8 cm B) 10 cm C) 12 cm D) 13 cm
31. What is the volume of a frustum of cone with height 5 cm and the radii of its bases are 3 cm and 4 cm?
A) $\frac{295}{3}\pi\ cm^3$ B) $\frac{240}{3}\pi\ cm^3$ C) $\frac{105}{3}\pi\ cm^3$ D) $\frac{290}{3}\pi\ cm^3$



32. If ice-cream consists of a hemisphere with radius 10 cm and a cone as shown in side figure, then its volume is ___________
A) $\frac{8000}{3}\pi\ cm^3$ B) $8000\pi\ cm^3$ C) $4000\pi\ cm^3$ D) $\frac{4000}{3}\pi\ cm^3$
33. A triangular prism has height 30cm. its base is a right triangle with legs 10cm and 24cm. The volume of this prism is _____________
A) $2000\ cm^3$ B) $3000\ cm^3$ C) $4000\ cm^3$ D) $6000\ cm^3$
34. A frustum of a regular square pyramid has height 5 cm. The upper base is of side 2 cm and the lower base is of side 6 cm. The lateral surface area of the frustum is ________
A) $16\sqrt{29}\ cm^2$ B) $32\sqrt{21}\ cm^2$ C) $16\sqrt{27}\ cm^2$ D) $32\sqrt{27}\ cm^2$
35. The lower base of the frustum of a regular pyramid is a square 4 cm long; the upper base is 3 cm long. If the slant height is 6 cm, then its lateral surface area is _______
A) $24\ cm^2$ B) $12\ cm^2$ C) $21\ cm^2$ D) $18\ cm^2$



36. If a right circular cylinder whose base radius is 10 cm and whose height is 12 cm is drilled a triangular prism hole whose base has edges 3 cm, 4 cm and 5 cm as shown below, then what is the total surface area of the remaining solid?
A) $(440\pi + 12)cm^2$ B) $(440\pi + 132)cm^2$
C) $(440\pi - 12)cm^2$ D) $(440\pi - 132)cm^2$

Appendix B. Sample Lesson Plan of EG1

General Information

Name of the School: S1 Name of teacher: T1
Grade: 10 Subject: Mathematics
No. of students: 34 Unit: Six
Duration: 45’ Lesson topic: Revision of Prisms
Date: _____________ Study Group: EG1

Specific Objectives: At the end of this lesson, students will be able to:

- Name different types of prisms.
- Explain the difference between right and oblique prisms.
- Identify the base, lateral faces, and altitude of a given prism.
- Define what they mean by the total surface areas and volume of prisms.
- Calculate the total surface area of prisms.
- Calculate the volume of prisms

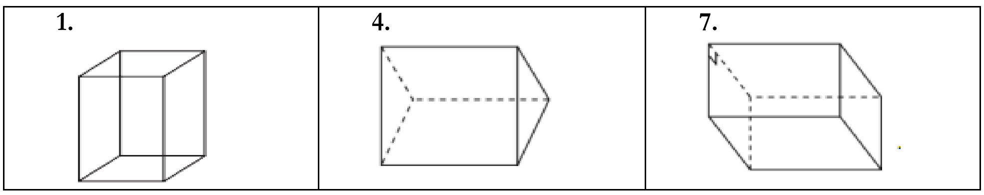
Teaching Method: GIBI-VT
Approach: Constructivist (5E instructional model)
Technique: Activity-Based
Teaching and Learning Materials: Models and pictures of prisms, and activity paper, and a ruler.
E1: Engagement (___Minutes)
The teacher will ask students to answer the following questions.

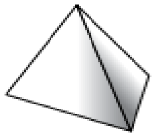
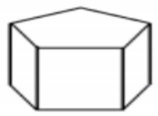
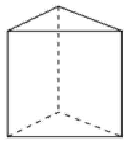
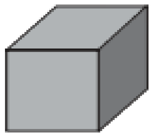
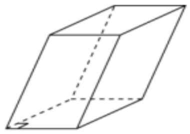
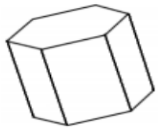
- What is mean by a prism?
- What is the area formula of a rectangle, parallelogram, and triangle?

Following this, the teacher gives an activity paper, which contains Figure B1 below, to each group and asks them to do the following activities and to express their justification.

1. Please circle the numbers that represent prism figures.
2. Classify the selected prisms into right and oblique prisms.
3. Write the name of each prism.

Figure B1
Collection of solid figures



2. 	5. 	8. 
3. 	6. 	9. 

The above activities contained the following variation theory concepts.

Objects of learning: To help students to:

- Identify prisms
- To classify prisms into right and oblique
- Name prisms

Critical features used

For activity 1, the critical features were:

- Being a solid figure with two congruent bases, and
- Its lateral faces are rectangles or parallelograms.

For activity 2, the critical feature was "where the foot of the altitude of the prism points the base (at its midpoint or somewhere else)?"

For activity 3, the critical features were

- What type of plain figure is its base (triangle, rectangle, pentagon, hexagon, -----)?
- Is the prism right or oblique?

Invariant: Being solid figures

Patterns of Variation Used

- **Fusion:** Figure B1 contains solid figures that are prisms, and some that are not.
- **Conceptual variation:** The prisms were presented in different orientations (i.e., using standard and non-standard position figures).

E2: Exploration (___Minutes)

1. What is meant by the total surface area of prisms?
2. Deriving the total surface area formula of a prism.

Procedure (Transforming solid figures into plane figures) *[Scaffolding]*

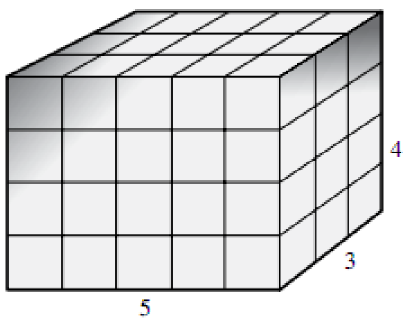
1. Take a rectangular prism.
2. Cut out all (the six) rectangular faces or planes.
3. Calculate the area of each rectangular surface.
4. Add these areas that provide the total surface area of a prism.
5. What is the total surface area formula of the prism? Does it work for all types of prisms? Why? [Prompts]

Activity II

1. What is meant by the volume of prisms?
2. Using the following model figure of a prism, derive its volume formula.
   [Hint: count the number of cubes inside the prism]

Figure B2

Rectangular Prism



3. What is the volume formula of the above prism? Does this formula work for an oblique prism? Why?

E3: Explanation (___Minutes)

- A prism is a polyhedron with two of whose faces are parallel polygons (bases), and whose remaining faces are Parallelograms (lateral (sides) faces). The distance between the two bases is h, which is measured along a line at right angles to both bases.

- A right prism is a prism whose lateral faces are rectangles. When a prism is a right rectangular one, all surfaces are rectangular. In the case of other ones, the non-rectangular faces are considered the bases,
- A rectangular solid (box) is a prism bounded by six rectangles. The length l, width w, and height h are its dimensions.
- A cube is a rectangular solid bounded by six squares. A cubic unit is a cube whose edge measures 1 unit.
- A parallelepiped is a prism bounded by six parallelograms. Hence, the rectangular solid and cube are special parallelepipeds.
- The surface area of any given object is the area or region occupied by the surface of the object.
- V = Area of the base × the height. V = l × w × h = Ab× h

E4: Elaboration (___Minutes)

1. What type of solid figure is the interior side of your classroom?
2. What shapes of plane sections does a right triangular (an oblique triangular) prism has?
3. What will be the shapes of the lateral faces of the prism if its bases are regular polygons? Are they congruent? Why?
4. Find the volume of a rectangular solid having a length of 6 cm, a width of 4 cm, and a height of 1cm.

E5: Evaluation (___Minutes)

1. What are the similarities and differences between a rectangular solid (box), a cube, and a parallelepiped?
2. What is the volume of a block of ice you can make in a rectangular container that's 12 cm long, 8 cm wide, and 10 cm high? (Disregard the fact that water expands at 4° C.)
3. Give homework from the students' textbook.

Appendix C. ANOVA assumptions test results

Table C1

Test of homogeneity of variances

	Levene Statistic	df1	df2	Sig.
MAPreTS	.611	2	101	.545

Note: MAPreTS = Mathematics Achievement Pre-Test Scores

Table C2

Shapiro-Wilk's tests of normality of MAPreTS

	Study Groups	Shapiro-Wilk Statistic	df	Sig.
MAPreTS	EG1	.964	31	.379
	EG2	.972	39	.427
	CG	.962	34	.280

Table C3

ANOVA results for MAPreTS

		Sum of Squares	df	Mean Square	F	Sig.
MAPreTS	Between Groups	251.393	2	125.697	12.816	.000
	Within Groups	990.597	101	9.808		
	Total	1241.990	103			

Appendix D. ANCOVA assumptions test results

Table D1

Shapiro-Wilk tests of normality for MAPostTS

		Shapiro-Wilk		
Variable	Study Groups	Statistic	df	Sig.
MAPostTS	EG1	.969	30	.507
	EG2	.957	37	.161
	CG	.947	32	.117

Table D2

Levene's test of equality of error variances for MAPostTS

F	df1	df2	Sig.
2.477	2	96	.089

a. Design: Intercept + MAPreTS + Groups

Table D3

Tests of between-subjects effects (interaction effect)

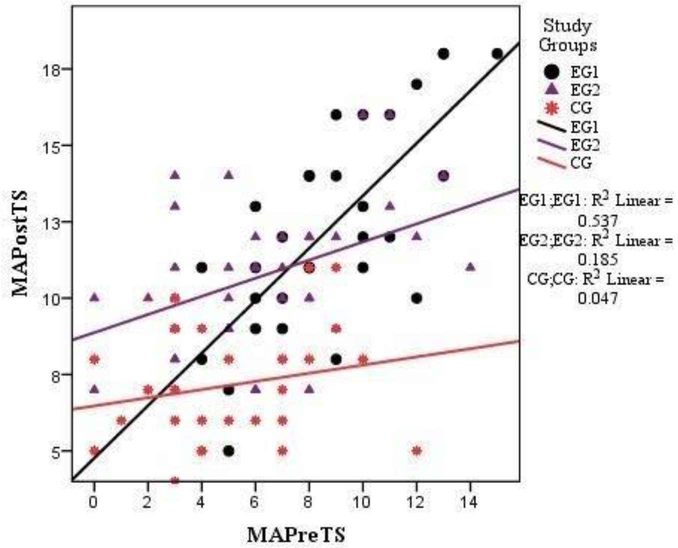
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	661.657	5	132.331	29.858	.000	.616
Intercept	685.519	1	685.519	154.673	.000	.625

Groups	43.842	2	21.921	4.946	.009	.096
MAPreTS	161.617	1	161.617	36.465	.000	.282
Groups * MAPreTS	74.928	2	37.464	8.453	.000	.154
Error	412.181	93	4.432			
Total	11054.000	99				
Corrected Total	1073.838	98				

Note: Dependent Variable: MAPostTS

Figure D1

Linear relationship MAPreTS and MAPostTS



Appendix E. MANOVA assumptions test results

Multicollinearity and singularity assumptions of MANOVA were checked by creating correlation matrices. As shown in **Table E1**, the Pearson correlation coefficient (r) values were between .428 and .61, indicating the non-violation of multicollinearity and singularity assumptions.

Table E1

Correlations among GO sub-constructs post-test scores

		MaGPostTS	PApPostTS	PAvPostTS
MaGPostTS	Pearson Correlation	1	.610**	.428**
	Sig. (2-tailed)		.000	.000
	Sum of Squares and Cross-products	641.373	355.706	203.863
	Covariance	6.350	3.522	2.018
	N	102	102	102

**. Correlation is significant at the 0.01 level (2-tailed).

Box's M result was used to examine the tenability of the homogeneity of variance-covariance assumption of MANOVA. The result in **Table E2** below Box's test M = 20.112, (F (12, 28809.996) = 1.6, p = .084), indicates that the homogeneity of variance covariance assumption wasn't violated.

Table E2

Test of equality of covariance matrices a for GO sub-constructs post-test scores

Box's M	20.112
F	1.601
df1	12
df2	43528.470
Sig.	.084

a. Design: Intercept + Groups

Levene's test for equality of variance results, as shown in **Table E3**, indicate that the p-values of the post-test scores of almost all GO sub-constructs are greater than 0.05, indicating the data have equal variances.

Table E3

Levene's test of equality of error variances for GO sub-constructs scores

	F	df1	df2	Sig.
MaGPostTS	3.289	2	99	.046
PApPostTS	3.178	2	99	.051
PAvPostTS	2.355	2	99	.100

a. Design: Intercept + Groups

Appendix F. Regression analysis assumptions test results

Table F1

Residuals statistics of MAPostTS

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	5.80	13.41	10.04	2.269	99
Std. Predicted Value	-1.869	1.485	.000	1.000	99
Residual	-7.879	5.973	.000	2.410	99
Std. Residual	-3.202	2.427	.000	.979	99
Stud. Residual	-3.293	2.476	.000	1.006	99
Deleted Residual	-8.336	6.216	-.002	2.541	99
Stud. Deleted Residual	-3.483	2.547	.000	1.019	99
Mahal. Distance	2.014	8.324	3.960	1.301	99
Cook's Distance	.000	.126	.011	.017	99
Centered Leverage Value	.021	.085	.040	.013	99

a. Dependent Variable: MAPostTS

Table F2

The Durbin-Watson value of MAPostTS

					Change Statistics					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change	Durbin-Watson
1	.446a	.199	.190	2.978	.199	24.048	1	97	.000	
2	.640b	.409	.397	2.570	.211	34.261	1	96	.000	
3	.667c	.445	.427	2.505	.036	6.084	1	95	.015	
4	.685d	.470	.447	2.461	.025	4.415	1	94	.038	1.933

a. Predictors: (Constant), GIBI-VT vs. TTM
b. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM
c. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM, PApPostTS
d. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM, PApPostTS, Student's Gender
e. Dependent Variable: MAPostTS

Table F3

The tolerance values of MAPostTS

		Unstandardized Coefficients		Standardized Coefficients			Collinearity Statistics	
Model		B	Std. Error	Beta	t	Sig.	Tolerance	VIF
1	(Constant)	9.072	.359		25.302	.000		
	GIBI-VT vs. TTM	3.194	.651	.446	4.904	.000	1.000	1.000
2	(Constant)	7.125	.454		15.682	.000		
	GIBI-VT vs. TTM	5.142	.653	.717	7.872	.000	.741	1.350
	GIBI alone vs. TTM	3.632	.620	.533	5.853	.000	.741	1.350
3	(Constant)	4.246	1.249		3.400	.001		
	GIBI-VT vs. TTM	4.839	.648	.675	7.465	.000	.714	1.401
	GIBI alone vs. TTM	3.542	.606	.520	5.848	.000	.738	1.355
	PApPostTS	.277	.112	.192	2.467	.015	.962	1.039
4	(Constant)	3.939	1.235		3.189	.002		
	GIBI-VT vs. TTM	4.897	.638	.683	7.682	.000	.713	1.403
	GIBI alone vs. TTM	3.663	.598	.538	6.126	.000	.731	1.368
	PApPostTS	.266	.110	.185	2.410	.018	.960	1.041
	Student's Gender	1.117	.532	.159	2.101	.038	.989	1.011

a. Dependent Variable: MAPostTS

Figure F1

Normality of MAPostTS residuals statistics

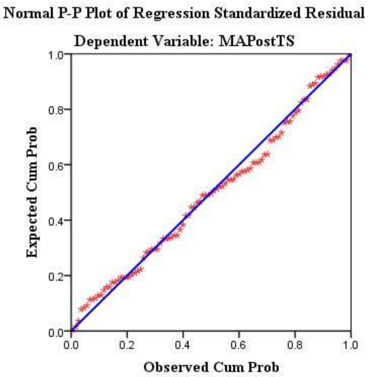


Figure F2

Homoscedasticity of residuals for MAPostTS

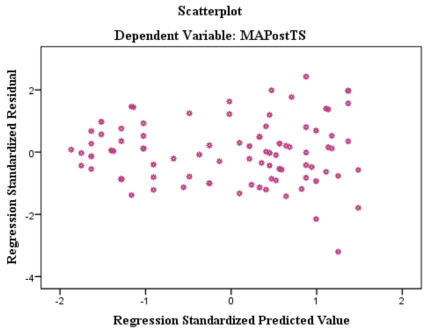


Table F4

ANOVA result of a significant R for MAPostTS

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	213.334	1	213.334	24.048	.000b
	Residual	860.504	97	8.871		
	Total	1073.838	98			
2	Regression	439.661	2	219.830	33.277	.000c
	Residual	634.177	96	6.606		
	Total	1073.838	98			
3	Regression	477.830	3	159.277	25.388	.000d
	Residual	596.008	95	6.274		
	Total	1073.838	98			
4	Regression	504.568	4	126.142	20.829	.000e
	Residual	569.270	94	6.056		
	Total	1073.838	98			

a. Dependent Variable: MAPostTS
b. Predictors: (Constant), GIBI-VT vs. TTM
c. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM
d. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM, PApPostTS
e. Predictors: (Constant), GIBI-VT vs. TTM, GIBI alone vs. TTM, PApPostTS, Student's Gender