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Research paper

Hyperbolic Geometry "Lupis": Hypothetical learning trajectory of the Triangle in the Context of Sumatra

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ABSTRACT

Lobachevsky geometry is an elective course that is difficult for students to learn. One of them is the concept of the number of angles in a triangle less than 180 degrees. The purpose of this study is to produce a design of a Triangle learning trajectory in Lobachevsky Geometry using the context of the Sumatran traditional snack "*lupis* cake". The subject of this study is a mathematics education student at one of the universities in Bengkulu province. The research approach used in this study is Design Research. This approach involves an iterative cycle consisting of three phases, namely the preparation phase, the experimental phase, and the retrospective analysis phase. The result of this study is that there are five activities in the learning trajectory of the Triangle in Lobachevsky Geometry using the context of Sumatran traditional snack "*lupis* cake". Using this context, students are able to find the concept of the number of angles on a triangle less than 180 degrees. The conclusion is that the design of the Triangle learning trajectory in Lobachevsky's Geometry in the context of the traditional Sumatran snack "*lupis* cake" is valid and practical for finding the number of angles in a triangle.

Keywords: hypothetical learning trajectory, triangle, Lobachevsky geometry, context of traditional Sumatran snacks, *lupis* cake

Hyperbolic geometry is known as Lobachevsky Geometry (Emil & Jenő, 2023). Such geometry is a non-Euclidean geometry that offers a new perspective on understanding hyperbolic spaces (Nugroho et al., 2022; Sukestiyarno et al., 2023). The concepts and principles in geometry are very difficult for most mathematics education students. Initial surveys found that undergraduate students of mathematics education in Bengkulu made mistakes in understanding Lobachevsky's geometry. It is a failure to understand Lobachevsky's parallel axioms and their implications. Students think that the sum of the angles of a triangle is always 180 degrees, even in Lobachevsky's geometry. They are confused between Euclidean straight lines and curved lines on hyperbolic geometry. Some have difficulty imagining the shape and properties of triangles in hyperbolic space. They rely on incorrect Euclidean intuition when solving Lobachevsky's geometry problems. Also, the difficulty in proving the theorems that exist in Lobachevsky's geometry is due to the difference in the existing axioms.

Based on the search, the cause of these students' mistakes is the lack of a strong understanding of the axioms and basic principles of Euclidean geometry, which are the basis for understanding non-Euclidean geometry. Lack of understanding of the deductive structure of non-Euclidean geometry (Nugroho et al., 2022). Students are unable to solve problems and prove theorems in Lobachevsky geometry (Herawaty et al., 2020). This is due to their misunderstanding of Lobachevsky geometry concepts. Furthermore, in our preliminary research, we found that mathematics curricula and education lack in-depth coverage of non-Euclidean geometry.

To overcome these errors, several efforts have been made, such as Increased understanding of concepts, through emphasis on a deep conceptual understanding of the axioms and basic principles of Lobachevsky geometry (Nugroho et al., 2021; Widada, Herawaty, Hudiria, et al., 2020c). Learning uses models and visualizations to help students understand abstract concepts (Waluyo et al., 2019). Provides many problem-solving exercises involving various concepts and theorems in Lobachevsky geometry (Widada, Agustina, et al., 2019; Widada, Herawaty, Hudiria, et al., 2020c). Thus, geometry software is useful for helping students explore Lobachevsky geometry. It can encourage students to discuss and collaborate in understanding difficult geometric concepts. Students are able to gain a deeper understanding of Lobachevsky geometry.

Although various efforts have been made to improve the understanding of the concepts and principles of triangles in Lobachevsky's geometry, undergraduate mathematics education students in Bengkulu have still not achieved this. Therefore, an appropriate alternative is needed to overcome students' errors in understanding the concepts and principles of Lobachevsky geometry, such as the sum of the angles in a triangle being less than 180 degrees (Beeson et al., 2015; Kyeremeh et al., 2023; Widada et al., 2020). The research results state that understanding of geometry can be improved if taught using local cultural contexts and an ethnomathematics approach (Kyeremeh et al., 2023; Stathopoulou et al., 2015; Sunzuma, 2022; Nugroho et al., 2019). Based on the findings of several studies, incorporating elements of local culture—such as traditional architectural forms (e.g., traditional houses), folk games, or patterns in batik art—can make geometry lessons much more relevant and meaningful for students. This approach serves as a foundation, allowing students to bridge the gap between concrete, everyday experiences and abstract geometric concepts. It is hoped that by transforming cultural realities into mathematical activities, students will have a better grasp of and be more interested in the subject of geometry (Abdullah et al., 2023; Fouze & Amit, 2021; Nugroho et al., 2022). Mathematical value in the building pattern of Toraja traditional houses studies the right geometry learning medium (Nugraha, 2019). Also research that discusses ethnomathematics in learning geometry based on mosque ornaments (Purniati et al., 2022).

Some research focuses on the development and use of ethnomathematics-based learning media, such as teaching aids inspired by local culture (Andriani et al., 2020). The use of these media can improve students' mathematical critical thinking skills (Indriani et al., 2024), as well as their motivation and activeness in learning (Novitasari et al., 2021; Wulandari et al., 2024). There is a beneficial impact from the application of geometry learning media rooted in ethnomathematics on critical thinking skills in mathematics Skills (Bintoro et al., 2021; Dasari & Agustiani, 2020; Indriani et al., 2024). It can be concluded that the ethnomathematics method significantly influences students' geometry learning achievements or achievements. Students who study with this approach tend to show an increased understanding of geometry concepts and problem-solving skills compared to students who study with a conventional approach (Nugroho et al., 2021). The implementation of the ethnomathematics approach is able to foster student learning motivation, because students feel closer to the material being studied. Learning that is connected to the local culture, increasing student interest (Widada, Herawaty, et al., 2020). These research findings demonstrate that an ethnomathematical approach (such as incorporating local culture) has significant potential to enhance students' understanding and interest in geometry. By connecting geometry with local culture, learning becomes more relevant, meaningful, and enjoyable for students. Thus, a local cultural approach can facilitate students' understanding of geometric concepts and principles. Cake *Lupis*, as a traditional snack rich in culture, can be used as a tool to explain the concept of triangles in Lobachevsky's geometry. That is as a recommendation from the results of the research (Nugroho et al., 2022; Sukestiyarno et al., 2023; Widada, Herawaty, Hudiria, et al., 2020). However, research related to triangles in non-Euclidean geometry in an ethnomathematical context is still very limited (Ma'rifah et al., 2019).

Research on non-Euclidean geometry through an ethnomathematics approach is limited because it has so far focused on Euclidean geometry. However, several studies have applied an ethnomathematics approach that has the potential to aid understanding of non-Euclidean geometry. This approach aims to visualize abstract concepts while emphasizing the use of local culture as a visual aid. This principle significantly aids students' understanding of non-Euclidean geometry (Sukestiyarno et al., 2023). Ethnomathematics serves as a bridge connecting non-Euclidean geometry with students' everyday experiences. Although non-Euclidean geometry may seem very abstract, its principles can actually be linked to concepts relevant to students' experiences. This includes ideas such as space, distance, and patterns. Thus, ethnomathematics can help students develop stronger geometric intuition (Ma'rifah et al., 2019). By exploring patterns and structures in local cultures, students can develop a deeper understanding of spatial relationships.

Based on this description, this study offers the context of the local culture of Bengkulu, namely the traditional snack " *Lupis* Cake", can be used as a starting point for learning triangles in Lobachevsky geometry (Ma'rifah et al., 2019; Nasriadi et al., 2022; Widada, Herawaty, Hudiria, et al., 2020). Empirically, students can find the sum of the angles in a triangle in that context is less than 180 degrees (Widada, Herawaty, Hudiria, et al., 2020). With the local cultural context in Lobachevsky Geometry in the local cultural context, learning can be more meaningful, relevant, and interesting for students. Therefore, it is necessary to increase research that combines ethnomathematics with Lobachevsky Geometry to create a more effective learning approach. This study proposes a learning trajectory design for triangle material in Lobachevsky Geometry by utilizing the context of traditional Bengkulu food, namely "*Lupis* Cake". The main problem formulation of this research is: How to design a learning trajectory for Triangle material in Lobachevsky Geometry using the context of traditional Sumatran food, "*Lupis* Cake"?

METHOD

This research is a validation study by applying the type of design research. This design research combines the development of educational materials with theoretical advancements, allowing for a comprehensive exploration of how students learn math. The focus is on validation studies on the design of hypothetical learning trajectories (HLTs) (Simon & Tzur, 2012; Simon et al., 2018). The design research process consists of three main phases: preparation and design, teaching experiments, and retrospective analysis. Each phase informs the next, creating a continuous cycle of improvement (Eerde, 2013). The hypothetical learning trajectory serves as an important tool during these phases, guiding the design and assessment of instructional strategies (Gatekeeper et al., 2016). The design research approach emphasizes the understanding of the cognitive processes of mathematics education students about the concept and principle of the pagan triangle of Lobachevsky geometry using traditional snacks "*Lupis* Cake". This is a snack sold in traditional markets in South Sumatra, Indonesia. Which is shaped like a triangle. Researchers adapt teaching methods based on real-time feedback from students (Preacher et al., 2015). It is important to compare the physical properties of the curvature of the *Lupis* Cake (extrinsic) and the geometry of hyperbolic space itself (intrinsic). In this context, the *Lupis* Cake serves as a physical model or cognitive framework to help students visualize how lines that appear "straight" to a curved surface intrinsically satisfy Lobachevsky's axioms.

The focus of this research is the validation of learning trajectory designs that allow the identification of effective strategies that enhance basic and complex mathematical knowledge (Žakelj, 2018). Design research is to validate HLT design in depth through the learning process. This study involved 15 mathematics education students of the University of Muhammadiyah Bengkulu and 15 mathematics education students at the University of PGRI Silampari Lubuk Linggau. Design research typically takes place in three different stages: preparation and design, teaching experiments, and retrospective analysis (Preacher et al., 2015).

Preparation and design

In the preparation and design stages, the researcher prepares the initial design of the hypothetical learning trajectory. There are three activities in this stage. First, the researchers provided students with a pretest problem related to the Lobachevsky triangle concept. The results of the pretest showed that students tended to have difficulty understanding the problem of the Lobachevsky triangle concept, some of them were confused by the concept of triangles in Euclid's geometry, and some had difficulty representing the Lobachevsky triangle concept visually or abstractly. Second, the researcher conducted a literature review of various ways of teaching about the concepts and principles of geometry. Based on this research, it was found that the teaching of the concept and principle of the Lobachevsky triangle uses the context of a traditional snack that is close to the student's mind, namely *Lupis* Cake. *Lupis* Cake is a traditional triangular-shaped snack. Third, the researcher chose this context as the starting point for students learning to discover the concept of the Lobachevsky triangle.

Based on the results of this initial stage, the researcher developed a hypothetical learning trajectory consisting of three components: learning objectives, learning activities, and hypothetical learning processes (Prediger et al., 2015; Gatekeeper, 2019). Hypothetical learning activities and student trajectories finding the cone volume formula are presented in **Table 1**.

Teaching experiments

The teaching trial stage is carried out as a prototype trial. This was an initial trial of the HLT design involving seven students studying in a group. The results of these tests are used as inputs to improve the design of HLT. The revision is about the readability of student worksheets, clarity of instruction and language, and the attractiveness and suitability of the local cultural context used in learning.

Table 1

Activities and HLT of the Lobachevsky Triangle Concept

Main Activities	Main objectives	Hypothetical activity
Activity-1: Identifying Problems with Local Cultural Contexts	Strengthening students' understanding of the Lobachevsky triangle concept using the context of Lupis Cake	Students identify problems with the concept of lobachevsky's triangle using the context of Lupis cake;
Activity-2: Problem Representation	Represents about the relationship between "Lupis" Cakes and triangles.	Students represent the problem of the lobachevsky triangle concept using the context of Lupis cakes;
Activity-3: Creating a Completion Plan	Students found a problem-solving plan about Lobachevsky's triangular concept using the context of a Lupis Cake.	Students make a problem-solving plan for the concept and principle of the lobachevsky triangle using the context of Lupis cakes;
Activity-4: Implementing the Plan	Train students' ability to solve unfamiliar triangle problems, as they are different from Euclid's triangles.	Students carry out problem-solving plans for the concept and principle of the lobachevsky triangle using the context of Lupis cakes;
Activity-5: Evaluating Problem-solving	Improve students' ability to evaluate and compare several other triangles.	Students evaluate the problem-solving of the concept and principle of the lobachevsky triangle using the context of Lupis cake;
Activity-6: Making Conclusions	Students draw conclusions about the concept and principle of the Lobachevsky triangle.	Students draw conclusions about the concept and principle of the lobachevsky triangle using the context of the Lupis cake formally.

Retrospective analysis

Retrospective analysis involves reviewing and evaluating the completed design process. In the retrospective analysis technique, interviews are conducted: Interviews, which are interviews with design team members, stakeholders, and users. Observation, which is observing the use of products or services resulting from the design process. Data Analysis, which is analyzing quantitative and qualitative data collected from various sources. Thus, retrospective analysis is a valuable tool for improving design practices and resulting in better designs.

RESULTS AND DISCUSSION

This study finds the actual learning trajectory of the concept and principle of triangles in Lobachevksy geometry. To find these concepts and principles, this study uses the context of traditional snacks that are very familiar in the people of Sumatra, especially Bengkulu. The snack is *a Lupis* Cake See **Figure 1**.

Figure 1

Lupis cake (traditional snack in Sumatra) (Source: Authors' own elaboration)

Figure 1 is an image of *glutinous rice Lupis* Cake which is one of the traditional snacks in Sumatra. These snacks are often sold in traditional markets in Bengkulu City and in Lubuk Linggau City. The community really likes the snack, Cake. Try to pay attention to *the Lupis* Cake that is visible from the front.

The problem given to students is "*Based on the length of the Lupis* Cake (Figure 1), *how do you find the triangle and its characteristics? Are you trying to identify the shape of the Lupis* Cake? *Represent the shape of the Lupis* Cake *in the picture on paper? Make a plan to find the triangle and its characteristics! Determine the characteristics of the triangle? What do you think of the characteristics of the triangle you acquired? Draw conclusions about the triangle and its characteristics!*"

Based on the results of student work and confirmed by in-depth interviews, it can be presented in actual learning trajectory activities. There are six student activities that can be presented in order as follows.

Activity-1: Identifying problems with local cultural contexts

In Activity-1, students can identify the problem of the number of angles in a triangle using the context of a *Lupis* Cake. The student identified the curved sides of the triangle in the *Lupis*, see Figure 2.

Figure 2
Identification of the outline segment "Lupis cake" (Source: Authors' own elaboration)

Based on Figure 2, it is confirmed through interviews between students and researchers. Snippets of interviews between researchers (R) and students (S1) about the observation of *Lupis* Cakes and their relationship with Lobachevsky Geometry. Snippets of the interview are as follows.

R: OK, have you observed *Lupis* Cakes, right? Can you tell us what you saw?

S1: Yes, sir. The *Lupis* Cake is triangular in shape, wrapped in banana leaves, and then sprinkled with brown sugar and sprinkled with coconut.

R: Great. Now, try to focus on the shape of the triangle. What did you notice?

S1: The shape... Unique, sir. Unlike a regular triangle. It looks like there are curves on the sides.

R: Exactly. The curve is interesting. In Euclidean Geometry (the usual geometry you learn in school), the sides of the triangle are straight. But, in Lobachevsky's Geometry, the sides of a triangle can be curved.

S1: Oh, so the indentation in *the Lupis* Cake is like the curved side in the Lobachevsky triangle, right?

R: Yes, that's right.

Based on Figure 2 and the interview snippet, through *Lupis* Cakes, students are able to identify problems. Students, visualize the concept of triangles with curved sides which is one of the hallmarks of Lobachevsky's Geometry.

Activity-2: Representation of triangle problems using the context of lupis cake

The specific student task in Activity 2 demonstrates that they are capable of representing the Lobachevsky triangle. Students accomplish this by sketching three distinct curved lines based precisely on the physical edges of the traditional Lupis cake. This exact representational process is clearly illustrated in Figure 3.

Figure 3
Visual representation of "Lupis' cake" (Source: Authors' own elaboration)

From Figure 3, the researcher confirmed through interviews between students and researchers. A more in-depth interview snippet of the representation of line segments on *a Lupis* Cake with curved sides, attributed to the Lobachevsky triangle. Snippets of the interview are as follows.

R: Try showing me the sketch you made.

S1: This, sir. I drew a triangle shape of a *Lupis* Cake and tried to mark the line segments on the edges.

R: What made you choose curved lines?

S1: Because the edges of *the Lupis* Cake are indeed curved, sir. So, I think the curved line is more appropriate to represent it.

R: Exactly. In Euclidean Geometry, we usually use straight lines to represent line segments. But, in Lobachevsky's Geometry, the line can be curved.

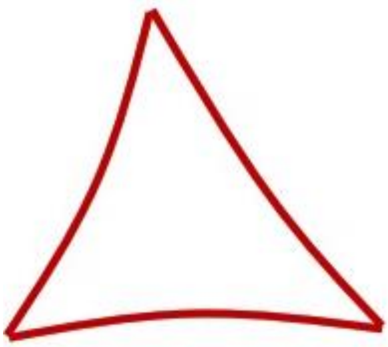
S1: So, the curved line segment in my sketch can be analogized to the line segment in the Lobachevsky triangle!

Based on the description above, students represent the curved line segment on the *Lupis* Cake as an analogy of the line segment in the Lobachevsky triangle. He was able to use the context of everyday life to represent the concept of triangles in Lobachevsky's geometry.

Activity-3: Creating a completion plan

Activity-3 is that students are able to develop a problem-solving plan. He planned to use flexible wire to form the curved sides. He planned to measure the angles in the triangle. This can be seen in Figure 4.

Figure 4
Triangle based on "Lupis' cake" (Source: Authors' own elaboration)



Based on Figure 4, the researcher confirmed through interviews. Snippet of an interview between the researcher (R) and a student (S1) about the preparation of a problem-solving plan by making the corners of the Lobachevsky triangle based on *a Lupis* Cake.

R: Yes, that's great. Try to tell us your ideas.

S1: First, I think we can use *the shape of the Lupis* Cake as the base of the triangle. Then, we need to find a way to represent the curved sides of the Lobachevsky triangle.

R: Good idea. How would you represent those curved sides?

S1: I think we can use a flexible rope or wire to form those curved sides. We can attach the rope or wire to a flat surface, following the shape of the edge of the *Lupis* Cake.

R: That's an interesting idea. Then, what about the corners?

S1: Yes, sir. Maybe we can make some variation of the shape of a triangle with different curved sides, and measure the angles. That way, we can see how the change in shape of the curved sides affects the number of angles in the triangle.

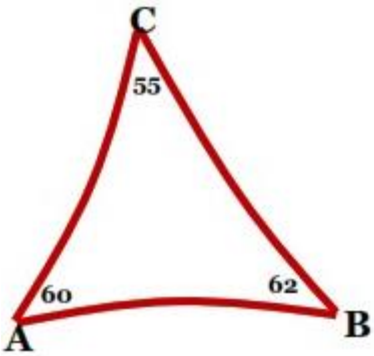
Students are able to use *Lupis* Cake as the base of the Lobachevsky triangle. He represented the curved sides with a rope or bending wire and planned to make measurements of the angles with a protractor.

Activity-4: Implementing the plan

Students carry out problem-solving plans are Activity 4. He conducted an experiment of measuring three angles on a triangle using a protractor. This activity can be seen in **Figure 5**.

Figure 5

Measurement of the triangle "Lupis' cake" (Source: Authors' own elaboration)



The paper-and-pencil activity was confirmed by an interview with the student. Snippets of interviews between researchers (R) and students (S1) about the measurement of the corners of curved side triangles based on *Lupis* Cakes. The footage is as follows.

R: "Good morning, S1. How did you measure the corners of the triangles of *the Lupis* Cake that you did?"

S1: "Good morning, sir. I have already taken measurements using a protractor. The results are quite interesting, sir."

R: "Interesting? Try to explain in more detail."

S1: "So, I measured the three corners on the *Lupis* Cake *triangle*. The first corner is about 55 degrees, the second corner is about 60 degrees, and the third corner is about 62 degrees."

R: "Then, what is the total number of the three corners?"

S1: "After I add up, the total is 177 degrees, sir."

R: "177 degrees? That means less than 180 degrees, right?"

S1: "That's right, sir. This corresponds to what we discussed earlier about Lobachevsky's Geometry. Whereas in Euclidean geometry, the sum of angles in a triangle is always 180 degrees. But, in Lobachevsky's Geometry, the number is less than 180 degrees.

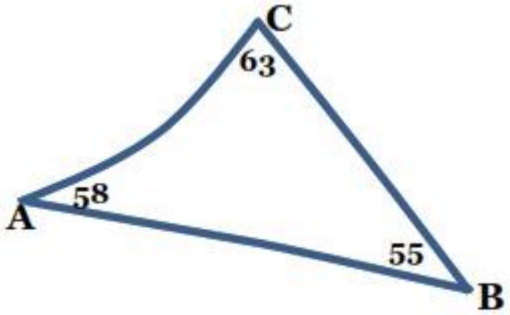
Based on **Figure 5** and interview snippets, students were able to measure the angles of *the triangles of Lupis* Cakes using a protractor. The results of the measurements showed that the sum of the three angles was less than 180 degrees. This means that the measurement results in the triangle correlating with the concept of Lobachevsky Geometry. This shows the importance of observing everyday objects in mathematics learning, especially geometry.

Activity 5: Students evaluate problem-solving

In Activity 5, students evaluate problem-solving by repeating the measurement of the corners of curved triangles with variations in the shape of *Lupis*, Cakes, and other Cakes. He tried to compare, one of the triangles and the size of the angle like **Figure 6**.

Figure 6

Another size of the triangle (Source: Authors' own elaboration)



Based on Figure 6, the researcher confirmed through interviews. Interview excerpts between researchers (R) and students (S1) about students evaluating problem-solving by repeating measurements of the corners of curved triangles with variations in the shape of *Lupis* Cakes.

R: "Okay! What is the result of repeating the measurement of the triangle corners of a *Lupis* Cake with different shape variations?"

S1: "Yes, sir. I've repeated the measurements with some *Lupis* Cakes that have different triangle shapes."

R: "That's great. Try telling me what you found."

S1: "I found that although the shape of the triangle of *the Lupis* Cake varies, the sum of the three corners is always less than 180 degrees, sir."

R: "It was a very interesting result. Can you give me an example of your measurements?"

S1: "Sure, sir. For example, on the first *Lupis* Cake, I got angles of 58 degrees, 63 degrees, and 55 degrees, for a total of 176 degrees. In the second *Lupis* Cake, the corners are 60 degrees, 59 degrees, and 61 degrees, for a total of 180 degrees. And in the third *Lupis* Cake, the angle is 57 degrees, 62 degrees, and 59 degrees, for a total of 178 degrees. "That is why

R: "So, even though there is a slight variation in the angle value, the trend remains consistent, i.e. the number of angles is less than 180 degrees."

S1: "That's right, sir. I also noticed that the more curved the sides of the *Lupis* Cake triangle, the smaller the number of corners."

Qualitatively, the discovery process is demonstrated in the following dialogue:

S1: "Initially, I thought the sum of the angles was 180 degrees, but when I placed the protractor on the curved surface of the *Lupis*, the lines weren't as straight as they were on flat paper."

S1: "It turns out the sum is only about 176 degrees because the path follows an inward curve."

Based on the interview snippet and Figure 6, students were able to evaluate the solution of the problem through the repetition of the measurement of the corners of the *triangles of the Lupis* Cake with variations in shape. The measurement results are consistent, i.e. the number of angles is less than 180 degrees. It was an observation of the relationship between the curvature of the sides and the number of angles, which was a reinforcement of the understanding of Lobachevsky's geometry through experiments.

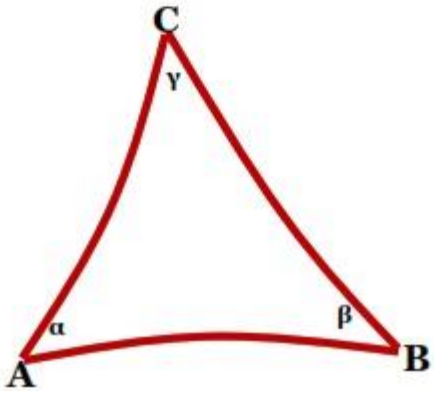
Here, students also use software. GeoGebra software was used to add a digital verification phase to complement the manual data and ensure scientific rigor. Students were instructed to model the same characteristics of the hyperbolic plane (the Poincaré Disc model) in GeoGebra after measuring the angles of triangles on a real *Lupis* Cake. Through this exercise, students were able to bridge the gap between formal mathematical software and real ethnomathematic objects by comparing their manual findings—angles that sum to less than 180°—with a dynamic digital simulation. Students' confidence in the validity of non-Euclidean geometric characteristics was greatly enhanced by this dual-approach proof.

Activity 6: Making conclusions

Students make conclusions in Activity-6. He made the statement mathematically by drawing a triangle formally which is the triangle ABC with three angles in α, β, and γ. In this activity, students make conclusions about a principle or nature of a triangle, namely $\alpha+\beta+\gamma<180^0$. An image of the ABC triangle can be seen in Figure 7.

Figure 7

Triangle formally (Source: Authors' own elaboration)



Based on these activities, it was confirmed by interviews. Snippets of interviews between researchers (R) and students (S1) about the deducation of the number of triangle angles in Lobachevsky Geometry based on the context of *Lupis* Cakes.

R: "OK. After the series of observations and measurements you have made, what conclusions can you draw about the number of triangle angles in Lobachevsky's Geometry, especially in the context of *Lupis* Cakes?"

S1: "Good afternoon, sir. Based on my observations, I concluded that the number of angles in a triangle with curved sides, as in *a Lupis* Cake, is always less than 180 degrees."

R: "Good. Can you state that conclusion in the form of a mathematical statement?"

S1: "Sure, sir. If we suppose the angles of the triangle ABC are α, β, and γ, then the mathematical statement is: $\alpha + \beta + \gamma < 180°$. (see Figure 7)"

R: "Exactly. So, through your observation of *the Lupis* Cake, you have succeeded in illustrating one of the hallmarks of Lobachevsky's Geometry."

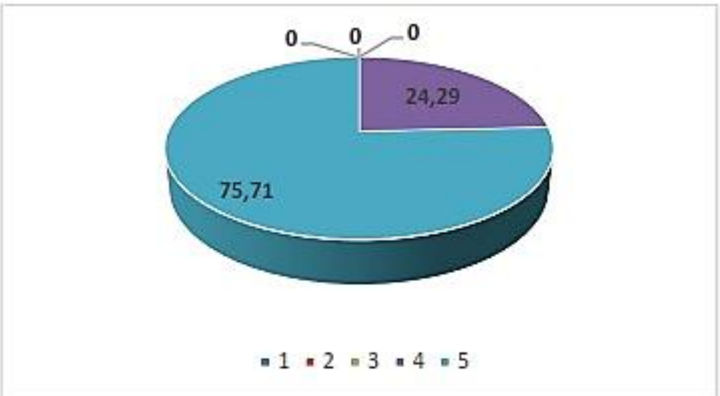
S1: "Yes, sir. I find it very helpful with the context of this *Lupis* Cake . The visualization of the curved triangle shape becomes easier to understand like Figure 7 as I did."

Based on Figure 7 and the confirmation, it shows that the students were able to conclude that the sum of the angles of the Lobachevsky triangle is less than 180 degrees. Also, it can make mathematical statements i.e. $\alpha + \beta + \gamma < 180°$. It is based on the use of *the analogy of a Lupis* Cake to explain the concept. This is how important it is to simplify mathematical concepts.

Furthermore, based on the comprehensive Hypothetical Learning Trajectory (HLT) validation data gathered during the preliminary design phase, the rigorous assessment conducted by seven academic expert validators can be systematically analyzed with the following results. These distinguished validators, comprising specialists in non-Euclidean geometry and ethnomathematics pedagogy, meticulously evaluated the construct validity, content relevance, and practical applicability of the instructional sequences. Their quantitative and qualitative feedback provided critical empirical foundations, ensuring that the culturally integrated Lupis cake model effectively supports the students' cognitive transition toward mastering the Lobachevsky triangle. The results can be seen in Figure 8-10 below:

Figure 8

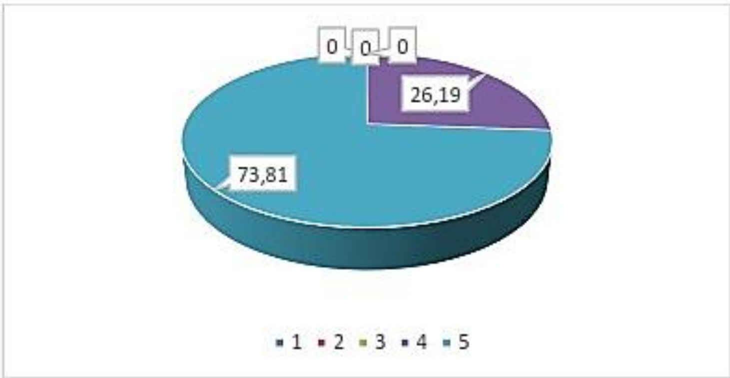
Percentage of HLT assessment by experts from material aspects (Source: Authors' own elaboration)



Based on Figure 8, the percentage of HLT assessments by experts from the content (material) aspect was obtained that 75.71% stated very valid, and 24.29% stated good, and 0% stated that it was sufficient. Thus, it is concluded that HLT from the aspect of categorized material is very valid.

Figure 9

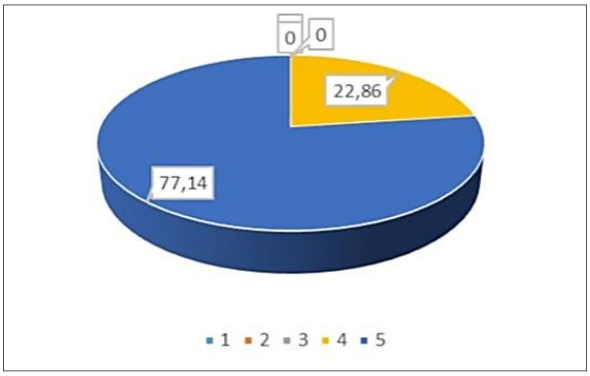
Percentage of HLT assessment by experts from the aspect of material presentation (Source: Authors' own elaboration)



Based on Figure 9, the percentage of HLT assessments by experts from the aspect of presenting the material was obtained that it was 73.81% who stated that it was very valid, and 16.19% stated good, and 0% who stated that it was enough to the bottom. Thus, it is concluded that HLT from the aspect of presenting classified material is very valid.

Figure 10

Percentage of HLT assessment by experts from language aspects (Source: Authors' own elaboration)

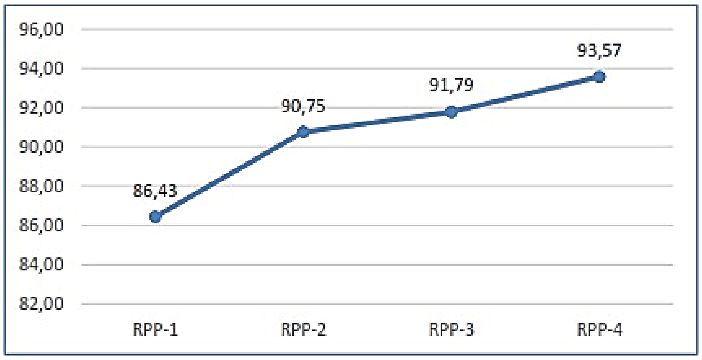


Based on Figure 10, the percentage of HLT assessments by experts from the language aspect was obtained that 10% stated very valid, and 22.86% stated good, and 0% stated that it was sufficient. Thus, it is concluded that HLT from the aspect of categorized language is very valid.

Based on the observation data on students during lobachevsky's geometry learning using the context of the traditional sumatran snack "*Lupis* Cake" in four meetings, the following diagram can be presented.

Figure 11

HLT implementation graph (Source: Authors' own elaboration)



Based on Figure 11, the implementation of the *learning trajectory* for the FD student group with an ethnomathematics approach is very well implemented. This is shown based on the implementation of *learning* *trajactory* which was implemented 86.43% in RPP-1, increasing continuously in the implementation of 90.75% in RPP-2; 91.79% in RPP-3, until the 4th meeting, namely 93.57% RPP-4 was carried out very well. This shows that the implementation of RPP from 1-4 has reached more than 85%, which means that the *learning trajectory* for the FD student group with an ethnomathematics approach is carried out very well. Thus, it is concluded that the design of the Triangle learning trajectory in Lobachevsky Geometry contextualized with the traditional Sumatran snack "*Lupis* Cake" is practical to facilitate the understanding of the concept.

The paired t-test results showed a significant increase in student understanding ($p < 0.05$). The effect size analysis showed a Cohen's d value of 0.85, indicating a high practical impact. The Shapiro-Wilk normality assumption was a prerequisite for this analysis. For the Hypothesis Test of the ability to understand the concept of triangles in Lobacehvsky geometry who learned using the context of the traditional ranks was higher than students who studied with conventional learning after controlling the influence of students' initial abilities. Experimental research data were analyzed and obtained from **Table 2**.

Table 2

Parameter Estimates

Parameters	B	Std. Error	t	Sig.
Intercept	37,271	3,866	9,64	0,000
X	0,503	0,080	6,27	0,000
A1B2	23,990	1,276	18,79	0,000

Dependent Variable: Ability to Understand Concepts

Based on **Table 2**, in the 3rd row, t count = 18.79 and p-value 0.000 < 0.05 mean that Ho is rejected. This means that the ability to understand the concept of triangles in Lobacehvsky geometry who learns using the context of the traditional hierarchy is higher than that of students who learn with conventional learning after controlling for the influence of students' initial abilities.

The results of this hypothesis test stated that the design of the Triangle learning trajectory in Lobachevsky Geometry contextualized with the traditional Sumatran snack "*Lupis* Cake" proved to be effective in facilitating the understanding of the concept. The learning mechanism through *Lupis* facilitates abstraction by transferring specific objects to schemas (Widada, Agustina, et al., 2019). The *Lupis* surface, a type of Poincaré disk, provides a visual clue that a "straight line" in hyperbolic geometry is an arc perpendicular to the boundary. This study follows previous research using non-Euclidean physical models but offers more opportunities for Indonesian students to use local artifacts (Dogutas, 2025; Phan, 2025).

The results of this study found that students can determine the concept of triangles in Lobachevsky geometry, which is a triangle whose sides are curved. Also, students are able to understand the principle of triangles in geometry. He made the statement that the sum of angles in a triangle is less than 180 degrees. The student states that the sum of all the angles in a triangle is less than 180^o. He formulated his mathematical statement as follows: For the triangle ABC with angles BAC = α, ABC = β, and ACB = γ, then $\alpha + \beta + \gamma < 180^o$. These findings support other research that the concept is very interesting, especially since it contradicts the common understanding in Euclid Geometry, where the sum of the angles of a triangle is always exactly 180^o (Widada, Herawaty, Hudiria, et al., 2020). It's important to understand the local cultural context used in this research. This serves as the starting point for learning Lobachevsky's geometry. In this geometry, Euclid's parallel postulate does not apply, resulting in the angle sums of triangles being different (Nugroho et al., 2022). Specific learning activities designed to introduce this concept to students (Widodo et al., 2025; Abakah & Brijlall, 2024; Widada, et al., 2019; Vuola, 2017). The use of visual or manipulative aids to help students understand these abstract concepts (Ibrahim et al., 2024).

A limitation of this study is the homogeneity of the subjects, which only included students in a specific region of Sumatra. To increase external validity, future research should apply this design to a broader and more diverse population. This is based on the idea that learning occurs within a specific educational context, so educational research needs to determine how “local instruction theory” can be applied in a broader global educational context (Gravemeijer & Cobb, 2006). Increasing sample diversity will help determine whether visual perception of ethnomathematics objects is influenced by geographic background or whether it is universal in understanding non-Euclidean geometry concepts (Qi & Terry, 2025). This study involved thirty mathematics students from Bengkulu and thirty from Lubuklinggau. While this is a good starting point for design research, generalization to life contexts outside Sumatra should be done with caution. Using a digital protractor with an accuracy of 0.1°, various individuals used a three-step measurement technique (technical triangulation) to ensure the accuracy of angle measurements on the physical *Lupis* object. A margin of error of ±0.5° was set to accommodate imperfections in the physical surface of the Cake. Another limitation relates to its reliance on a specific local cultural instrument (*Lupis* Cake), which may not be applicable in other areas (Halpern et al., 2025). The physical validity of manual food testing is more susceptible to variability than computer-based mathematical models.

The results of this study indicate that students actually have the potential to master the concepts and principles of non-Euclidean geometry, even though this material is often considered difficult. This study emphasizes the importance of introducing students to various types of geometry, not just Euclidean geometry. This research provides insights into how to design effective learning to teach more complex geometric concepts and principles (Schefers, 2026). Understanding Lobachevsky geometry helps develop students' critical thinking and problem-solving skills. It also helps students understand the diversity of mathematics and how mathematical concepts can be applied in various contexts. Therefore, this study provides invaluable information for the world of mathematics education. Thus, the learning trajectory design for "Triangles in Lobachevsky Geometry," using the context of the traditional Sumatran food, "*Lupis* Cake," is valid and practical for finding the sum of the angles of a triangle.

CONCLUSION

This study confirms that students have the capacity to internalize the concepts of Lobachevsky Geometry, in particular the understanding that the sum of the angles of a triangle can be less than 180 degrees. Through the design of the context-based learning trajectory '*Lupis*' Cake, we have validated an effective and practical approach to bridge the gap of understanding towards non-Euclidean concepts that were previously considered difficult.

Based on these findings, we make some suggestions: There is a need for further development of teaching materials that integrate local cultural contexts in mathematics learning, especially for non-Euclidean geometry. It is important to improve teacher training on how to teach non-Euclidean geometry and how to effectively use local cultural contexts in the learning process. Adequate learning resources and tools must be provided to support the learning of non-Euclidean geometry. Further research is needed to find a more interesting and effective learning medium for Lobachevsky Geometry.

The implications of this research are: This research proves that non-Euclidean geometry concepts can be taught to students using relevant and engaging contexts. The use of local cultural contexts can increase students' motivation and engagement in learning mathematics. This research contributes to the development of geometry and mathematics learning in general, making it more meaningful and relevant to students' daily lives. The research results support innovation in mathematics teaching approaches, particularly in the field of geometry. For practical implementation, it is recommended to start with non-Euclidean geometry by investigating local physical objects before moving on to formal analysis. This adaptive strategy challenges the identification of geometric structures in local architecture or traditional cuisine as a cognitive challenge.

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Ethical statement

This research was conducted in accordance with the guidelines of the declaration of helsinki and was approved by the ethics committee of the Mathematics Education Doctoral Program, Universitas Negeri Semarang, Indonesia, with code number: 026082025s3pmat, on august 26, 2025

Competing interests

The authors declare no conflict of interest.

Author contributions

Scolastika Mariani: Conceptualization, Data curation, Writing - original draft, Resources, Methodology, Visualization, and Project administration; Abdurrobbil Falaq Dwi Anggoro: Conceptualization, Data curation, Writing - original draft, Methodology, Formal analysis, Writing - review & editing, Validation, Supervision,

Investigation; Wardono: Methodology, Writing - review & editing, Supervision, Investigation, and Formal analysis; Bambang Eko Susilo: Resources, Writing - review & editing, Project administration, Visualization

Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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The authors acknowledge the use of ChatGPT to support the English translation of this manuscript. The command used included "Translate the following text into academic English." The results of this command were used to translate sections of the manuscript, which were then reviewed and edited by the authors. While the authors acknowledge the use of AI, they nevertheless declare that they, Scolastika Mariani, Abdurrobbil Falaq Dwi Anggoro, Wardono, and Bambang Eko Susilo, are the sole authors of this article and take full responsibility for its content, as outlined in the COPE recommendations.

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