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Research paper

# Mapping Students' Cognitive Load Profiles in Solving Geometric Integral Problems

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## ABSTRACT

Designing cognitively accessible mathematics instruction requires ensuring that learners can equitably process, understand, and apply complex mathematical ideas. This study examines students' cognitive load when solving geometric integral problems by analyzing intrinsic, extraneous, and germane load within the framework of Cognitive Load Theory. Using a descriptive qualitative approach, data were collected from 144 university students in mathematics-related programs at a public university in Indonesia through think-aloud protocols, written solutions, and classroom observations. The data were analyzed using NVivo-assisted thematic coding and supported by radar visualizations to strengthen analytical trustworthiness. The findings indicate that intrinsic load constituted the dominant cognitive burden, primarily arising from symbolic-visual confusion and visual-spatial difficulties when students attempted to coordinate algebraic expressions with geometric representations. Extraneous load further increased cognitive demands due to ambiguous verbal instructions, misinterpretation of symbolic cues, and excessive technical language; however, visual scaffolding and self-generated sketches were found to reduce unnecessary processing. Germane load emerged among a smaller group of students who demonstrated conceptual transfer, reflective verification, and schema integration, indicating the development of mathematical reasoning as students connected geometric structures with their corresponding integral representations. Overall, the study highlights the central role of instructional design in regulating cognitive load to support meaningful conceptual understanding and reasoning in solving visually complex mathematical problems.

**Keywords:** cognitive load theory, integral geometry, mathematical reasoning, visual-symbolic representation, cognitive load profiles

Solving integral geometry problems poses significant cognitive challenges for university students, particularly when they must simultaneously coordinate visual, symbolic, and spatial representations. Cognitive processing serves as the fundamental basis for understanding and internalizing mathematical concepts that are abstract and complex in nature (Adams et al., 2017; Cirino et al., 2015). This process requires the engagement of various components of working memory to process information, construct conceptual schemas, and generalize the mathematical principles being learned. To understand these challenges, Cognitive Load Theory (CLT) provides a valuable framework functioning in this study as an analytical lens to interpret and categorize students' cognitive processes, rather than merely a theoretical starting point to explain how the limited capacity of human working memory can influence learning effectiveness (Leppink, 2017; Sweller, 2023; Osman et al., 2026).

CLT emphasizes the importance of maintaining a balance among intrinsic load (the inherent complexity of the material), extraneous load (the manner in which information is presented), and germane load (the effort devoted to schema construction) to ensure optimal learning. Integral geometry is uniquely demanding because it requires students to manage three interconnected cognitive challenges: (1) high element interactivity coordinating multiple components such as radius, diameter, function equations, and integral boundaries simultaneously; (2) visual-symbolic conflict translating between algebraic expressions and geometric shapes, which often involves resolving discrepancies between symbolic notation and visual forms; and (3) mental imagery demands constructing and manipulating three-dimensional mental representations without physical referents. These cognitive demands exceed those of purely procedural or algebraic tasks and require the integration of multiple representational formats (Hawes & Ansari, 2020; Lowrie et al., 2019; Mix, 2019; Al-Hinai et al., 2026). In this regard, understanding how students experience, manage, and manifest their cognitive load becomes crucial for designing more effective pedagogical interventions. Therefore, to fully grasp the contextual application of cognitive load theory, it is important to examine how the theory operates in domains that demand complex representations, such as integral geometry.

Problems in integral geometry possess unique characteristics because they require thinking processes that involve the simultaneous integration of spatial and symbolic representations (Hawes & Ansari, 2020; Lowrie et al., 2019; Mix, 2019). When solving this type of problem, students must be able to visualize geometric forms in three dimensions, understand relationships among variables, and then transform these into symbolic and procedural integral expressions. This complexity gives rise to various types of cognitive load consistent with CLT: intrinsic load emerges due to the difficulty level of the material and the number of informational elements to be processed; extraneous load arises from suboptimal information presentation or confusing representations; while germane load relates to the students' effort to construct and strengthen conceptual schemas through reflection and the integration of prior learning experiences (Bolkan, 2016; Çakiroğlu & Bilgi, 2024; Huang, 2018; Klepsch et al., 2017; Klepsch & Seufert, 2020; Schuessler et al., 2025; Wang et al., 2022). When these three types of load appear simultaneously in a visual-symbolic context, students often encounter challenges in allocating cognitive resources efficiently. This condition highlights the importance of understanding how variations in cognitive load emerge and are distributed across individuals when they engage with integral geometry problems, so that distinct cognitive profiles can be identified as a basis for developing more adaptive learning strategies.

A growing body of research has demonstrated that Cognitive Load Theory (CLT) makes a significant contribution to the effectiveness of mathematics learning, particularly in problem-solving contexts that involve both visual and symbolic representations. Pictorial representations in simple arithmetic tasks do not always facilitate students' understanding, as they may even increase cognitive load when not properly integrated (Lindner, 2020; Scharinger, 2024). These findings align with those of Lowrie et al. (2019), Schweitzer and Brown (2007), and Yenilmez & Kakmaci (2015), who emphasize that visualizations not designed with cognitive principles in mind can raise total cognitive load and reduce learning performance. Meanwhile, research by Kosko (2022) shows that transforming visual representations into symbolic forms requires substantial germane load to support deeper structural understanding of mathematical concepts (Pape et al., 2022; Schulz, 2024). In the context of geometry learning, Jaelani (2021) and Özçakir et al. (2022) highlight that visual-reasoning tasks can stimulate the construction of conceptual meaning, yet they may also increase intrinsic load when presented to inexperienced problem solvers (Evagorou et al., 2015; Jeannotte & Kieran, 2017; Sterner et al., 2020; Osman et al., 2026). Most of these studies employ experimental and quantitative approaches to measure the effects of instructional design on performance and cognitive load, resulting in findings that tend to be macro-level and generalized. However, such approaches have not yet been able to uncover how cognitive load is subjectively experienced and how it varies across individuals in real problem-solving situations. Nevertheless, while these findings strengthen the position of CLT in mathematics education, there remains an underexplored gap that warrants further investigation (Korbach et al., 2018; Skulmowski & Xu, 2022).

Although research on cognitive load in mathematics education has grown rapidly, most previous studies have remained focused on quantitative measurements using numerical scales or perception-based questionnaires. While such instruments can provide a general overview of the level of cognitive load experienced by students, they are not yet capable of revealing the underlying thinking processes that give rise to that load in a deeper and more nuanced way (Naismith et al., 2015; Schmeck et al., 2015; Schuessler et al., 2025). The central research gap, therefore, is that quantitative instruments can measure perceived cognitive load levels, but they cannot capture how cognitive load emerges, fluctuates, and interacts with students' thinking strategies in real time during problem-solving. Quantitative approaches tend to reduce cognitive complexity into average scores, thereby losing the dynamic context that occurs as students interact with visual and symbolic representations during problem solving. In contrast, qualitative approaches based on thematic coding allow researchers to trace students' cognitive pathways in a richer manner through the analysis of narratives, verbal expressions, and the problem-solving strategies they employ. Unfortunately, such explorations remain very limited in cognitive load research, particularly in mathematics, a domain that requires the integration of spatial and symbolic reasoning (Akin & Murrell-Jones, 2018). Therefore, the use of NVivo-based thematic analysis becomes relevant for identifying, categorizing, and visualizing the cognitive themes that naturally emerge in students' thinking processes when they work through complex integral geometry problems.

The context of integral geometry education at the university level represents an area that demands a high degree of mathematical reasoning and visualization. Based on our collective teaching observations over 12 years of experience teaching calculus and geometry at university level, and further supported by preliminary pilot studies with 30 students, we have consistently noted that even high-achieving students struggle to connect integral procedures with geometric representations a difficulty that manifests in their problem-solving protocols and reflective accounts. Many students struggle to connect integral concepts with geometric representations because the thinking processes required to translate between visual, symbolic, and spatial forms involve complex cognitive coordination (Chang et al., 2016; Mainali, 2021; McGee & Moore-Russo, 2014). These difficulties often stem from limitations in visualizing volumes and areas generated by mathematical functions, which in turn increase intrinsic load and reduce problem-solving efficiency (Jojo, 2018; Tatar & Zengin, 2016). This gap in understanding indicates that instructional design at the university level must consider how students process, integrate, and manage visual and symbolic information simultaneously. Thus, an analytical approach is needed one that not only identifies the level of cognitive load but also captures its variations within the context of the representations and thinking strategies employed by students. Radar-based thematic visualization becomes relevant, as it is capable of representing differences in cognitive profiles across individuals comprehensively, illustrating the interrelationships among cognitive load components, and providing a holistic view of how students navigate the complexity of integral geometry (Börner et al., 2016; Marriott et al., 2021; Williamson, 2016). In line with these needs, this study proposes a new methodological approach.

This study introduces a methodological innovation by integrating NVivo-based thematic coding with radar visualization to map students' cognitive load profiles when solving integral geometry problems. The novelty and added value of this approach lie not in its individual components, but in their integration. First, the qualitative cognitive tracking via NVivo allows us to move beyond measurement to interpretation, systematically coding think-aloud protocols to uncover how and why specific cognitive loads emerge from the interaction between the learner and the task's representational demands. Second, the radar visualization serves as a powerful comparative visual model, transforming complex qualitative themes into a structured format that reveals the proportional intensity and interplay of different load types across individuals (Börner et al., 2016; Marriott et al., 2021; Williamson, 2016). This dual-pronged approach creates a comprehensive "cognitive profile" a nuanced, empirically grounded map of the student experience that is invisible to conventional quantitative scales. By integrating these methods, this study provides the first detailed exploration, to our knowledge, of how symbolic-visual confusion, instructional ambiguities, and conceptual transfer interact to shape the cognitive landscape of students grappling with the complexities of integral geometry.

The primary focus of this study is to map students' cognitive load profiles in solving integral geometry problems through an NVivo-based thematic analysis combined with radar visualization. This research seeks to identify how various types of cognitive load intrinsic, extraneous, and germane emerge, interact, and vary among students with different levels of problem-solving ability. The findings are expected to enrich the development of cognitive load theory by providing a more contextual, empirically grounded perspective based on authentic learning experiences. In addition, the results of this study are anticipated to broaden the application of visual-based instructional design that takes individual cognitive characteristics into account, thereby making the mathematics learning process more personalized and efficient.

## METHODS

### Research design

This study employs a descriptive qualitative approach supported by quantitative visual analysis. This approach was chosen because the aim of the research is not merely to measure the level of students' cognitive load but to gain an in-depth understanding of how that load is formed, interacts, and varies within the context of solving integral geometry problems, which demand both spatial and symbolic coordination (Gregg & Steinberg, 2017). By 'measure,' we refer to the systematic identification and thematic mapping of cognitive load manifestations through qualitative coding, where frequency counts in NVivo serve as indicators of thematic prominence rather than absolute measurements of cognitive load magnitude. The radar visualization thus illustrates comparative profile patterns, not psychometric scores. The qualitative model enables an exploration of students' thinking processes through NVivo-based thematic analysis, while the results are visualized using radar diagrams to illustrate differences in the intensity of the various types of cognitive load. The selection of this approach is grounded in Creswell's (2007) view that qualitative studies are effective for uncovering subjective experiences and complex cognitive patterns, whereas the integration of numerical visualization enhances the clarity of thematic interpretation.

**Participants**

This section describes the demographic characteristics of the participants involved in the study. Understanding participants' backgrounds is essential, as it may influence the way they process information, construct visual representations, and manage cognitive load when solving integral geometry problems. The study involved students from various academic programs with a mathematical orientation, taking into account the diversity of university locations and levels of academic achievement (GPA). All participants had successfully completed the prerequisite courses of Basic Calculus and Analytical Geometry, ensuring they possessed the foundational knowledge required to engage with the problem-solving task. This information provides a strong foundation for interpreting the contextual variations in cognitive load profiles across individuals.
The detailed background of the study participants is presented in **Table 1** below.

**Table 1**
*Background of participants*

| Category | Subcategory / Description | n | % |
|---|---|---|---|
| Total Participants | - | 144 | 100 |
| Study Programme | Education Programme (Mathematics) | 48 | 33.3 |
| | Mathematics Science | 52 | 36.1 |
| | Other Disciplines | 44 | 30.6 |
| Gender | Female | 79 | 54.9 |
| | Male | 65 | 45.1 |
| University Location | Urban Area | 102 | 70.8 |
| | Rural Area | 42 | 29.2 |
| Cumulative GPA Range | 3.51 - 4.00 (High) | 58 | 40.3 |
| | 3.01 - 3.50 (Moderate) | 61 | 42.4 |
| | ≤ 3.00 (Low) | 25 | 17.3 |

A total of 144 students participated in this study, representing various academic programs related to mathematics from universities located in urban (70.8%) and non-urban (29.2%) areas in eastern Indonesia. Based on gender distribution, there were 79 female students (54.9%) and 65 male students (45.1%). In terms of academic achievement, 40.3% of the participants had a high GPA (3.51-4.00), 42.4% were in the medium category (3.01-3.50), and 17.3% had a low GPA (≤ 3.00).

Inclusion criteria required that participants had: (1) completed Calculus II (integral calculus) with a minimum grade of C+, (2) completed Analytic Geometry with a minimum grade of C+, and (3) passed a brief content knowledge screening assessing recall of basic integral formulas and circle geometry concepts. These criteria ensured comparable academic exposure to the prerequisite knowledge for the integral geometry task and minimized the risk that cognitive load variations would stem from mismatched academic experiences rather than task characteristics.

All participants provided written solutions to the problem, which were used for initial screening and contextual triangulation. However, to enable the depth of analysis required for the think-aloud protocol and subsequent NVivo coding, a purposive subsample of 35 students was selected for in-depth cognitive analysis. This subsample was chosen to ensure maximum variation across GPA levels (high, medium, low), study programs, and university locations, thereby capturing a diverse range of cognitive processes representative of the larger sample.

Participant selection was carried out using a purposive sampling technique, taking into account the relevance of their academic backgrounds and learning experiences to the topic of integral geometry (Ames et al., 2019). The sample size of 144 participants was considered sufficient to capture diverse cognitive load profiles. Thematic saturation was achieved after analyzing approximately 100 participants, with subsequent data confirming and enriching existing thematic patterns rather than generating new categories. Thus, adequacy was determined by data depth and thematic variation rather than statistical representativeness.

While participant characteristics (major, GPA, university location) are presented descriptively, we conducted comparative analyses across GPA categories to examine whether cognitive load profiles varied systematically. Radar visualizations were used to compare profile patterns between high-GPA (3.51-4.00), moderate-GPA (3.01-3.50), and low-GPA (≤ 3.00) groups. No systematic differences were observed based on university location or gender, suggesting that cognitive load variations in this study relate more to individual processing strategies than demographic factors. Where patterns emerged (e.g., GPA differences in germane load indicators), these are reported in the Results section.

All participants took part voluntarily, and the study was conducted in accordance with ethical principles of educational research.

### Instruments

The task design in this study was developed to deeply explore students' thinking processes in solving contextual integral geometry problems that integrate visual, symbolic, and conceptual representations. The task was presented in the form of a word problem, as shown in **Table 2** below:

**Table 2**
*Task design*

| Task 1 |
|---|
| An architect is designing a window in the shape of a semicircle with a diameter of 4 meters. He wants to determine the area of the glass located in the upper-right quarter of the semicircle, as that portion will be fitted with special glass.<br>If the semicircle is modeled by the function<br>$$y = \sqrt{4 - x^2}$$<br>then the area of the region in question can be computed using the integral<br>$$\int_0^2 \sqrt{4 - x^2}\, dx$$<br>Questions:<br>Draw the region described in the Cartesian coordinate plane to help understand the problem.<br>Without directly computing the integral, explain how the area of the region can be determined using the concept of the area of a circle.<br>Compute the area of the region using an integral to verify the visual estimation. |

This task functions as an exploratory stimulus, not a representative measurement instrument. It is designed to elicit rich cognitive processes that reveal how students manage visual-symbolic integration when solving integral geometry problems. To ensure that observed cognitive load profiles were not merely task-specific, we triangulated findings through two strategies: (1) think-aloud protocols, which revealed consistent reasoning patterns across multiple problem attempts and confirmed that students employed similar strategies when encountering comparable problems; and (2) member-checking interviews conducted with 20 participants, who confirmed that their problem-solving approaches and cognitive difficulties reflected their typical strategies when facing similar visual-symbolic tasks. Thus, the cognitive load profiles obtained represent task-elicited patterns within a class of visual-symbolic problems, not responses unique to this single task.

All participants were asked to submit their work, even if they were unable to complete the problem, so that the researchers could identify which parts of the task were perceived as most difficult or induced the highest cognitive load. This step was designed to capture students' thinking processes comprehensively from understanding the geometric context, constructing visual representations, and converting them into symbolic form, to reflecting on the calculation results. Consequently, the data collected not only represent the final outcomes but also reveal cognitive strategies, conceptual errors, and levels of reflection that emerged throughout the problem-solving process.

The design of this task was developed based on six principles of mathematical problem construction to optimally stimulate students' thinking processes within the context of Cognitive Load Theory (CLT) (Richland et al., 2017). These principles are as follows:

1. The task involves additive or multiplicative relationships among three quantities radius, diameter, and the area of a circle which require relational understanding within a geometric context.
2. The task encourages students to analyze the relationships among these quantities rather than merely performing numerical substitution, thereby demanding coordination between conceptual and procedural knowledge.
3. The task does not contain explicit questions that can be answered directly; instead, it is designed to guide students toward reflective thinking through visual interpretation and geometric reasoning.
4. Students are trained to analyze the problem situation based on the information provided, particularly by connecting the representation of the function $y = \sqrt{4 - x^2}$ to the shape of a circle in the coordinate plane.

5. The language used is simple and familiar, allowing students to focus on mathematical reasoning without being burdened by linguistic complexity.
6. The problem situation integrates visual representations that align with students' ways of thinking, emphasizing the relationships among graph shapes, area regions, and integral symbols.

These six principles served as the main guidelines in the development, implementation, and analysis of the task. Through these principles, the problem functions not only as an assessment tool but also as a stimulus for observing students' cognitive dynamics when engaging with the relationship between geometric forms and integral mathematical models.

The primary instrument in this study consisted of an integral geometry problem, as presented above, designed to stimulate the three main types of cognitive load in Cognitive Load Theory, namely:

1. Intrinsic Load, which arises when students must connect the concepts of radius, diameter, and the circle function with the integral of an area;
2. Extraneous Load, which emerges from the complexity of the visual presentation and the need for symbolic interpretation;
3. Germane Load, which arises when students link their conceptual understanding with reflection and verification of results through two approaches (geometric and integral).

The content validity of the instrument was evaluated by three experts: a specialist in applied mathematics, a mathematics education expert, and an educational psychology expert. The assessed aspects included conceptual clarity, contextual accuracy, appropriateness of visual representations, and alignment with cognitive load indicators. The validation results showed an Aiken's V value of 0.89, which falls within the highly valid category, while the inter-rater reliability for the problem-solving strategy observation sheet yielded a Cohen's Kappa coefficient of 0.87, indicating a very high level of consistency among raters.

**Research procedures**

The research procedure was carried out through four main stages designed systematically to ensure the validity of both data collection and data analysis processes. In the preparation stage, the researchers developed the research instruments, which included an integral geometry problem-solving task, a brief interview guide, and an observation sheet for students' cognitive strategies. These instruments were validated by three experts in applied mathematics, mathematics education, and educational psychology to ensure conceptual accuracy, contextual relevance, and the instrument's potential to elicit cognitive load. This stage also included a limited pilot test to evaluate language clarity and the appropriateness of the task difficulty level for the target participants.

In the implementation stage, all 144 participants were given the integral geometry problem as described in **Table 2**. The task was completed individually within 60 minutes in a controlled classroom environment. All participants submitted written solutions, consisting of sketches, symbolic calculations, and step-by-step reasoning. However, to enable the depth of analysis required for this study, the think-aloud protocol was conducted only with the purposively selected subsample of 35 students. This subsample completed the task while verbalizing their thought processes, which were audio-recorded. The remaining 109 students completed the task under the same conditions but without the think-aloud requirement, contributing only written data. This approach allowed the researchers to obtain a direct and authentic depiction of students' thinking processes from the subsample, while the written data from all participants provided a broader context for triangulation.

The next stage involved documentation and transcription, during which all student work and audio recordings were transcribed verbatim to preserve the integrity of the cognitive data generated. Each transcript was then imported and coded using NVivo 14, which served to systematically organize the data and support the thematic analysis process. The coding procedures were carried out through three main phases open coding, axial coding, and thematic coding to identify indicators of intrinsic, extraneous, and germane cognitive load that emerged during the problem-solving process.

The final stage was verification and triangulation, aimed at ensuring the reliability and credibility of the analytical results. This process was conducted through inter-coder agreement, in which two independent researchers cross-checked the coding outcomes to assess the consistency of interpretations across coders. In addition, source triangulation was performed by comparing verbal data, written work, and observational notes on cognitive strategies. The final thematic analysis results were then visualized using a radar diagram (spider-web chart) to depict the distribution, intensity, and variation of cognitive load profiles across individuals. This visualization enabled the identification of dominant patterns and differences among student profiles, providing an empirical basis for developing mathematics instructional designs that are adaptive to learners' cognitive characteristics.

**Data analysis**

Data analysis in this study was conducted using an NVivo-based thematic approach to identify, organize, and interpret cognitive patterns that emerged from the process of solving integral geometry problems. The analysis followed three main coding stages open coding, axial coding, and selective/thematic coding each aimed at systematically and comprehensively mapping students' cognitive load profiles.

In the open coding stage, the researchers identified units of meaning emerging from the verbal transcripts and students' recorded thinking processes. Each segment of data whether spontaneous utterances from the think-aloud protocol or written strategies on the worksheet was coded into basic categories such as "indicators of confusion," "spontaneous visualization strategies," "attempts at symbolic modeling," or "conceptual reflection." This process enabled the researchers to recognize early indicators of cognitive load experienced by students naturally during the solution of integral geometry problems.

The next stage, axial coding, was conducted to group the open codes into higher-level conceptual categories based on shared meaning and logical relationships among ideas. At this stage, several categories emerged, including "symbolic representation with heavy load," "reflective visual strategies," "graph interpretation errors," and "efforts to construct conceptual schemas." The relationships among these categories were analyzed to identify causal patterns between the types of representations students used and the levels of cognitive load that emerged.

In the selective or thematic coding stage, the researchers organized and formulated the main themes representing the three dimensions of cognitive load as described in Cognitive Load Theory namely intrinsic load, extraneous load, and germane load. These themes were developed based on students' thinking tendencies and the characteristics of the strategies used by each individual. For example, students who demonstrated difficulty in understanding relationships among variables were categorized under high intrinsic load, whereas students who experienced difficulties due to ineffective visual representations were categorized under high extraneous load. Conversely, students who were able to reflectively connect integral concepts with geometric structures were categorized as having a dominant germane load, indicating effective construction of conceptual schemas.

After the coding process was completed, the verified thematic analysis results were visualized using a radar diagram (spider-web chart) to illustrate the patterns of dominance and intensity among the types of cognitive load within the student population. Each axis in the radar diagram represents one category of cognitive load, while the area covered on the chart indicates the relative intensity experienced by individuals or groups. This approach allows for a combination of qualitative analytical depth and rich visual interpretation, providing not only a deeper understanding of the variations in students' cognitive profiles but also results that are communicative and easy for academic readers to interpret.

Thus, this data analysis process provides a robust framework for mapping the relationships between students' representational strategies and the cognitive load profiles that emerge during the solution of integral geometry problems. The integration of NVivo-based thematic analysis and radar visualization strengthens the reliability of the interpretations while supporting the primary aim of the study: to uncover patterns of individual and comparative variation in cognitive load within the context of visual-based mathematics learning.

## RESULTS

Comparative analysis across GPA categories revealed systematic patterns: high-GPA students (3.51-4.00) showed relatively higher frequencies in germane load indicators (Conceptual Transfer, f = 9; Reflective Verification, f = 6), whereas low-GPA students (≤ 3.00) exhibited predominantly intrinsic and extraneous loads (Symbolic-Visual Confusion, f = 15; Ambiguous Verbal Instruction, f = 11). No systematic differences were observed based on university location or gender, indicating that cognitive load variations in this study relate more to individual processing strategies and prior achievement levels than demographic factors. This section elaborates on the characteristics and interrelationships among these load types, supported by thematic coding from NVivo analysis and quantitative mapping through radar visualization. The overall findings suggest that intrinsic and extraneous loads were dominant, reflecting the high complexity of visual-symbolic integration required by the problem, while germane load appeared selectively among students who demonstrated reflective verification and conceptual transfer. Before discussing each theme in detail, Table 3 presents an overview of the frequency distribution of cognitive load categories, serving as the basis for subsequent thematic interpretation.

Figure 1 provides a multidimensional view of the proportional intensity of cognitive load categories derived from NVivo coding. The blue, orange, and green zones correspond respectively to Intrinsic, Extraneous, and Germane Loads, allowing for a simultaneous comparison across the nine subdimensions.

The figure clearly shows that Intrinsic Load and Extraneous Load occupy the largest cognitive bandwidths particularly the nodes Symbolic-Visual Confusion (f = 38) and Ambiguous Verbal Instruction (f = 26). These high frequencies indicate that students faced substantial cognitive demands when interpreting the mathematical model $y = \sqrt{4 - x^2}$ and visualizing the semi-circular area without explicit scaffolding.

In contrast, the Germane Load section, although smaller in magnitude, reflects productive cognitive engagement. Nodes such as Conceptual Transfer (f = 14) and Reflective Verification (f = 9) represent instances where students internalized the link between geometric area and integral representation, demonstrating higher-order reasoning consistent with schema integration processes described by (Sweller, 2020, 2023).

Overall, the radar visualization emphasizes a polarized cognitive pattern: students invested significant cognitive resources in decoding task complexity (intrinsic) and managing unclear representations (extraneous), whereas only a subset succeeded in transforming that effort into meaningful conceptual understanding (germane). This imbalance highlights the need for visual-symbolic scaffolding and structured instructional design to reduce unnecessary load while fostering deep comprehension.

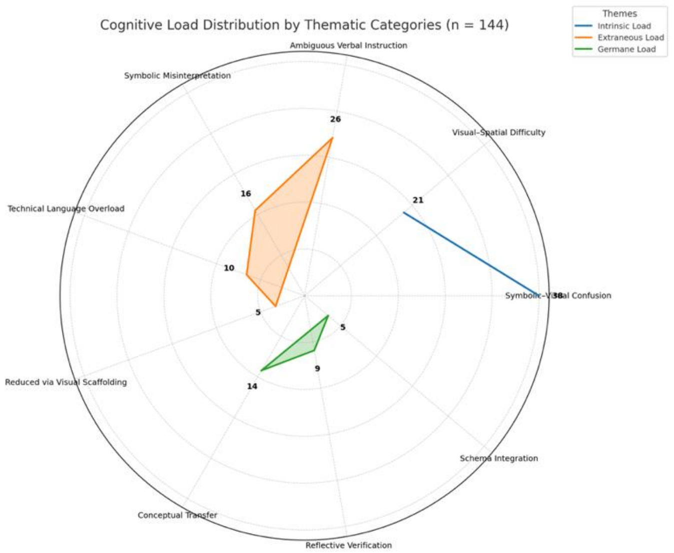
**Table 3**

*Frequency distribution of cognitive load categories (n = 144)*

| Thematic Category | Axial Coding | Frequency (f) | Percentage (%) | Interpretation |
|---|---|---|---|---|
| Intrinsic Load | Symbolic-Visual Confusion | 38 | 26.4 | High intrinsic complexity due to dual representation (symbolic & geometric). |
| | Visual-Spatial Difficulty | 21 | 14.6 | Struggles in transforming mental imagery into spatial representation. |
| Extraneous Load | Ambiguous Verbal Instruction | 26 | 18.1 | Misinterpretation of task wording without visual cues. |
| | Symbolic Misinterpretation | 16 | 11.1 | Errors linking algebraic symbols to graphical form. |
| | Technical Language Overload | 10 | 6.9 | Overburden from complex mathematical terms. |
| | Reduced via Visual Scaffolding | 5 | 3.5 | Improved clarity when visual aids were introduced. |
| Germane Load | Conceptual Transfer | 14 | 9.7 | Evidence of schema connection between geometry and integral concept. |
| | Reflective Verification | 9 | 6.3 | Students validated results using multiple representations. |
| | Schema Integration | 5 | 3.5 | Formation of integrated conceptual understanding. |
| Total | | 144 | 100% | |

**Figure 1**

*Radar visualization of students' cognitive load distribution across thematic categories*



**Intrinsic load analysis**

The analysis of intrinsic cognitive load demonstrates that students' major cognitive challenge stems from the inherent complexity of the mathematical representation itself. Solving the problem of finding the area of a quarter of a semicircular window modeled $y = \sqrt{4 - x^2}$ required the simultaneous coordination of three distinct forms of knowledge: symbolic manipulation, geometric visualization, and conceptual reasoning. This multi-representational integration generates a high element interactivity, the central determinant of intrinsic load (Ngu et al., 2018; Weurlander et al., 2017). Students were not merely expected to perform algebraic operations but also to construct mental imagery that connects the symbolic equation to the geometric form of a semicircle. Such integration overloaded the working memory of many participants, as shown by the dominance of symbolic-visual confusion (f = 38) and visual-spatial difficulty (f = 21) in the NVivo coding output.

A large proportion of students experienced cognitive friction when attempting to link the algebraic expression with its geometric counterpart. Participant S12 described this phenomenon vividly:

*"At first, I didn't realize that the function was part of a circle. I was just solving it like a regular square-root equation, but the question was about a window shape, and that confused me."*

This response illustrates that intrinsic load does not only emerge from the complexity of the content but from the disconnection between symbolic reasoning and visual meaning. Students often processed the equation as an isolated symbolic task rather than as a representational model of a real geometric object (Cañadas et al., 2018; Jupri & Sispiyati, 2020; Papadopoulos, 2019).

The second major pattern, visual-spatial difficulty, was associated with students' struggle to locate or imagine the quarter-region of the semicircle in the coordinate plane. Participant S37 reflected:

*"I tried to draw the graph, but I couldn't figure out which part of the semicircle was meant. I understood the radius was 2, but where exactly was the area we had to find?"*

This comment shows that spatial reasoning demands a mental transformation of the function into a visual image, which places additional demands on working memory. Intrinsic load becomes particularly high when learners must integrate abstract information across different modalities without sufficient prior schema (Albus & Seufert, 2023; Leahy & Sweller, 2016). The absence of guided visualization or reference models in this task exacerbated students' mental load, as they had to simultaneously recall, imagine, and manipulate spatial information to interpret the integral boundaries.

Another recurrent source of intrinsic strain involved the interpretation of the integration limits (0 to 2) and their relationship to the quarter of the semicircle. Several participants misinterpreted the numerical boundaries as arbitrary rather than geometrically meaningful. Participant S74 noted:

*"I thought 0 and 2 were just limits for the integral, like in any calculus question, not realizing that they define the right side of the semicircle."*

This misconception highlights how intrinsic load increases when procedural knowledge is detached from conceptual context. The integration limits, which should function as conceptual anchors, became isolated symbols disconnected from the underlying geometry. This phenomenon aligns with Cognitive Theory of Multimedia Learning, which posits that understanding occurs only when verbal-symbolic input is meaningfully linked with corresponding visual representation (Çeken & Taşkın, 2022; Désiron et al., 2025; Mayer, 2024). In this case, the absence of explicit scaffolding caused students' intrinsic load to exceed their cognitive capacity for integration.

Interestingly, a small group of participants managed to transform intrinsic load into germane processing by employing visual strategies to externalize mental representations. Participant S108 expressed this adaptive approach:

*"When I drew the half circle and shaded the top-right area, everything suddenly made sense. Then I understood why the limit was from 0 to 2."*

This illustrates that intrinsic load, while inherent to task complexity, can be mitigated through the construction of external representations that offload working memory. Visualization acts as a cognitive scaffold that allows learners to transfer cognitive effort from decoding to understanding (Dunn & Risko, 2016; Weis & Wiese, 2019). Students who visualized the function effectively could reallocate cognitive resources toward deeper conceptualization rather than struggling with isolated symbolic forms.

In summary, the Intrinsic Load Analysis reveals that students' cognitive difficulty is rooted in the need to integrate symbolic, spatial, and conceptual information simultaneously. The prevalence of symbolic-visual confusion and spatial disorientation indicates that intrinsic load was the most dominant type in the problem-solving process. These findings reaffirm that intrinsic load cannot be entirely reduced since it arises from the structure of the content itself but it can be managed through targeted scaffolding. Integrating explicit visual cues, guided representations, and step-by-step mapping between function and geometry would enable learners to redistribute cognitive resources more effectively. Such instructional alignment transforms intrinsic complexity into an opportunity for germane learning, supporting schema construction and meaningful mathematical understanding.

**Extraneous load analysis**

The second major dimension of cognitive load identified in this study was extraneous load, which emerged predominantly from instructional and representational inefficiencies. Unlike intrinsic load, which is inherent to the content itself, extraneous load originates from how information is presented and processed (Skulmowski & Xu, 2022). The NVivo thematic coding revealed four recurrent subcategories: ambiguous verbal instruction (f=26), symbolic misinterpretation (f=16), technical language overload (f=10), and reduction through visual scaffolding (f = 5). Collectively, these patterns accounted for 39.6% of all coded references, indicating that the presentation and linguistic framing of the tasksignificantly influenced the students' cognitive processing burden.

A dominant source of extraneous load came from unclear verbal or textual phrasing within the problem statement. Many students found it difficult to interpret expressions such as "the quarter of the semicircle on the right-upper part" without a supporting visual cue. Participant S15 shared:

*"The question said 'right-upper quarter,' but I wasn't sure which part that meant. I had to guess and check by drawing it several times."*

This finding underscores a fundamental principle of Cognitive Load Theory: when learners expend working memory resources to interpret ambiguous instructions rather than to process conceptual meaning, extraneous load increases (Bolkan, 2016). The problem's wording, though mathematically accurate, lacked the cognitive signaling needed to direct students' attention efficiently, forcing them to infer spatial meaning through trial and error.

Another key contributor to extraneous load was symbolic misinterpretation, where students struggled to connect algebraic notation to visual representation. Participant S29 reflected:

*"I didn't connect the function to the semicircle; I thought it was just another equation to integrate. I didn't notice it had a geometric meaning until the teacher explained."*

This indicates that symbolic abstraction when presented without multimodal support becomes cognitively taxing, as students must simultaneously decode syntax and recall geometric properties. This type of extraneous load arises when instructional materials fail to integrate corresponding verbal and visual information, causing learners to split their attention between unrelated cognitive streams (Guzmán & Zambrano R, 2024; Schroeder & Cenkci, 2018). This "split-attention effect" reduces working memory efficiency and impedes schema formation.

Linguistic complexity also played a considerable role. Technical terms such as "model function," "area under the curve," or "verification through integration" imposed additional cognitive strain, especially among students unfamiliar with such terminology. Participant S66 mentioned:

*"Some words in the question sounded too formal; I had to ask what 'model function' and 'verification' meant before solving it."*

This illustrates a language-based extraneous load, where comprehension difficulty arises not from conceptual demands but from vocabulary unfamiliarity (Njiku, 2025). When students allocate mental effort to decipher wording rather than relationships between mathematical concepts, cognitive efficiency decreases sharply. In this context, simplifying or contextualizing mathematical language could significantly reduce unnecessary load without diminishing the intellectual rigor of the problem.

Interestingly, a subset of students demonstrated that visual scaffolding could effectively reduce extraneous load. When diagrams or step-by-step illustrations were introduced, learners were able to redirect their cognitive resources from interpretation to reasoning. Participant S103 described this improvement succinctly:

*"Once I drew the semicircle and marked the quarter area, the problem became easy. I could focus on calculating instead of figuring out what the question meant."*

This aligns with redundancy reduction principles, which emphasize that clear visual representations eliminate redundant cognitive processes (Sweller, 2023). By externalizing the structure of the problem, learners no longer need to mentally reconstruct the geometry, thereby freeing working memory for conceptual integration and problem solving.

In summary, the Extraneous Load Analysis highlights that much of the cognitive strain experienced by students did not stem from mathematical complexity but from the inefficient communication and presentation of the problem. Ambiguous language, disjointed symbolic information, and technical jargon collectively inflated extraneous load, diverting cognitive energy away from understanding and reasoning. However, when supported by visual scaffolding either self-generated or teacher-provided students could redirect their mental focus toward conceptual understanding and verification. These findings reaffirm (Sweller, 2020) assertion that instructional clarity and representational coherence are central to minimizing extraneous cognitive load, particularly in visually demanding mathematical tasks.

**Germane load analysis**

The third and most pedagogically significant dimension of cognitive load observed in this study was germane load, representing students' active cognitive engagement in schema construction, conceptual transfer, and reflective reasoning. Unlike intrinsic and extraneous load, germane load does not impose additional burden on working memory; instead, it reflects the productive investment of cognitive effort toward understanding and meaning-making (Leahy & Sweller, 2016; Sweller, 2020, 2023). Although germane load accounted for only 19.5% of all coded instances, its presence signified deeper comprehension among students who successfully connected symbolic, visual, and procedural representations of the geometric integral task. Three subthemes emerged from the NVivo coding: conceptual transfer (f = 14), reflective verification (f = 9), and schema integration (f = 5).

The first subtheme, conceptual transfer, appeared among students capable of linking the symbolic form of the integral with its geometric interpretation. These students moved beyond rote manipulation by recognizing that the integral $\int_1^2 4 - x^2$ represented a quarter of a circular area with radius 2. Participant S24 described this moment of realization:

*"When I remembered that the formula $y = \sqrt{4 - x^2}$ is from a circle, I could imagine the shape. Then the integral became like finding a part of the circle's area, not just solving an equation."*

This reflection shows that germane load emerges when learners successfully transfer prior conceptual knowledge into a new context transforming procedural activity into meaningful reasoning. Such transfer exemplifies schema activation, in which existing mental structures are reorganized to accommodate new information, thus reducing long-term cognitive effort (Bossé et al., 2019; Stewart & Schmidt, 2017; Subanji & Supratman, 2015).

A second pattern of germane load involved reflective verification, where students deliberately checked consistency between their integral result and the geometric concept. Participant S52 shared:

*"After integrating, I compared my answer with the area formula $\frac{1}{4}\pi r^2$. It matched, so I knew my reasoning was correct."*

This metacognitive behavior reflects an essential form of germane processing learners consciously evaluating and reconciling multiple representations. Such reflection-driven processing converts short-term working memory activity into stable conceptual schema (Risko & Dunn, 2015). Students engaging in this process not only verified numerical accuracy but also reinforced their understanding of the equivalence between symbolic computation and geometric reasoning.

The third and deepest expression of germane load was schema integration, found among a smaller subset of students who could articulate how different aspects of the task (symbolic equation, visual model, and integral operation) were interrelated. Participant S118 explained:

*"Now I understand that the graph, the formula, and the integral are the same idea, just in different forms. Once I saw that, the question felt simple."*

This statement illustrates how germane load facilitates representational fluency the ability to shift between multiple modes of representation seamlessly (Ceuppens et al., 2018). Students demonstrating schema integration no longer experienced intrinsic load as an obstacle but as a framework for deeper reasoning. The integration of multiple cognitive elements suggests that learners had internalized an abstract schema capable of supporting future problem-solving beyond this specific task.

Notably, the development of germane load was often supported by self-initiated visualization or peer dialogue, where students externalized their reasoning and verbalized conceptual connections. Participant S93 recalled:

*"When discussing with my classmate, we compared the shape and the integral. Talking about it helped me understand the logic, not just the steps."*

This observation reinforces the role of social-(Cai & Gu, 2019). Collaborative explanation reduces extraneous load by distributing cognitive effort, allowing each learner to focus on integrating meaning rather than decoding information. In line with (Subanji & Supratman, 2015) interactive-constructive framework, dialogic interaction stimulates elaboration and facilitates long-term retention.

In summary, the Germane Load Analysis reveals that productive cognitive engagement manifested through conceptual transfer, reflective verification, and schema integration serves as the foundation for meaningful mathematical understanding. While only a minority of students achieved this level of processing, their strategies exemplify how germane load can transform complex problem solving into a process of conceptual reconstruction. Pedagogically, these findings emphasize the importance of fostering reflective dialogue, guided visualization, and representational translation activities in mathematics instruction. Through such interventions, teachers can not only reduce unnecessary cognitive load but also cultivate germane cognitive effort turning mental struggle into deep learning and durable understanding (see **Table 4**).

**Table 4**

*Summary of research findings*

| Cognitive Load Dimension | Thematic Categories (Axial Codes) | Key Findings / Evidence | Illustrative Student Quotes | Pedagogical Implications |
|---|---|---|---|---|
| Intrinsic Load | Symbolic-Visual Confusion<br>Visual-Spatial Difficulty | Students struggled to connect symbolic representation $y = \sqrt{4 - x^2}$ with its geometric meaning as a semicircle.<br>High element interactivity between symbolic, spatial, and conceptual information caused working memory overload. | *"I didn't realize that the function was part of a circle." (S12)*<br>*"I couldn't figure out which part of the semicircle was meant." (S37)* | Provide explicit symbolic-visual mapping and guided modeling to externalize abstract relationships; use visual scaffolds to manage element interactivity. |
| Extraneous Load | Ambiguous Verbal Instruction<br>Symbolic Misinterpretation<br>Technical Language Overload<br>Reduction via Visual Scaffolding | Cognitive burden increased due to unclear phrasing ("right-upper quarter"), formal mathematical jargon, and lack of diagrams. Some students reduced extraneous load by sketching the problem or receiving visual guidance. | *"I had to guess which part was the 'right-upper quarter.'" (S15)*<br>*"Once I drew the semicircle, the problem became easy." (S103)* | Simplify linguistic structure, integrate verbal and visual information, and apply dual-codingto prevent split-attention effect; provide contextualized examples. |
| Germane Load | Conceptual Transfer<br>Reflective Verification<br>Schema Integration | Productive cognitive effort emerged among students who linked the integral result to the geometric area.<br>Reflection and visualization enabled schema formation and conceptual understanding. | *"The integral became like finding a part of the circle's area." (S24)*<br>*"I compared my result with $\frac{1}{4}\pi r^2$ to check accuracy." (S52)*<br>*"The graph, the formula, and the integral are the same idea." (S118)* | Encourage reflective verification, peer explanation, and visual reasoning tasks to stimulate schema construction; transform cognitive struggle into meaningful engagement. |
| Integrated Interaction | Cross-theme synthesis | Intrinsic and extraneous loads were dominant (≈80%), but germane load represented deeper learning among reflective students.<br>Effective instructional design mediates transition from overload → understanding. | *"Discussing and sketching made the logic clearer, not just the steps." (S93)* | Design cognitive load balanced instruction: maintain challenge (intrinsic), reduce redundancy (extraneous), and promote reflection (germane). |

## DISCUSSION

The NVivo thematic analysis revealed three distinct cognitive load profiles among the 144 participants, visualized through radar patterns showing differential intensities across intrinsic, extraneous, and germane load categories. The most prominent finding was the dominance of 'Symbolic-Visual Confusion' (f = 38, 26.4%) and 'Ambiguous Verbal Instruction' (f = 26, 18.1%) in the thematic coding, as shown in Table 3 and Figure 1. These empirical patterns indicate that students' cognitive resources were disproportionately allocated to decoding task representations rather than constructing mathematical meaning. The radar visualization in Figure 1 further illustrates this imbalance, with the blue zone (intrinsic load) and orange zone (extraneous load) occupying substantially larger areas than the green zone (germane load). These findings provide the empirical foundation for interpreting students' cognitive load experiences through the lens of Cognitive Load Theory (Haji et al., 2015; Lee, 2019; Sweller, 2023), which posits that learning effectiveness is determined not only by the difficulty level of the material but also by how instructional design and visual representations regulate the distribution of students' cognitive resources (Bucur & Daskalova, 2020).

The first finding, grounded in the NVivo coding patterns, shows that intrinsic cognitive load arises from the complexity of tasks requiring the simultaneous coordination of symbolic, spatial, and conceptual representations. Coded indicators such as 'cannot relate function to circle' (n = 22) and 'confused about which area' (n = 20) appeared in 42% of transcripts, reflecting students' difficulties in connecting the equation $y = \sqrt{4 - x^2}$ with the visual representation of a semicircle and understanding the geometric meaning of integral boundaries. As Participant S37 articulated: "I understood the radius was 2, but where exactly was the area we had to find?" These empirical patterns the simultaneous coordination of symbolic equation, geometric shape, and spatial region exemplify what CLT terms high element interactivity (Ngu et al., 2018; Scharinger, 2024; Natsi & Vitsou, 2025), in which the number of elements that must be processed concurrently exceeds working memory capacity. The thematic frequency of these indicators (f = 59 combined for symbolic-visual and spatial difficulties) confirms that intrinsic load in this context is not merely content difficulty but the interconnectedness of multiple knowledge elements that require integration across representational formats.

The second finding, derived from both verbal protocols and visual error analysis, reveals that extraneous cognitive load emerged as an additional factor heightening information processing difficulty. The split-attention effect was evident in coded indicators such as 'text without picture confused me' (n = 18) and 'had to read instruction repeatedly' (n = 14), which fell under the thematic category 'Ambiguous Verbal Instruction' (f = 26, 18.1%). Students' visual errors documented in the written work and coded as 'misplacing the quarter-region' (f = 12) and 'drawing incomplete semicircles' (f = 9) provided behavioral evidence that the separation of textual instructions from visual representation increased extraneous processing demands. As Participant S45 explained: "I read 'upper-right quarter' many times but couldn't picture it until I drew it myself." This phenomenon reinforces the split-attention effect and redundancy effect (Swanson, 2016; Swanson et al., 2015; Schroeder & Cenkci, 2018; Houdyshell & Sanabria, 2025), whereby the lack of integrated visual cues caused students to expend working memory resources on interpreting the problem context rather than processing the underlying concepts. As the radar visualization in **Figure 1** illustrates, the 'Ambiguous Verbal Instruction' node occupied a substantial portion (18.1%) of students' cognitive profiles, confirming that presentation format significantly influenced cognitive load distribution. In addition, the use of technical terminology such as "model function" and "integral verification" increased the linguistic load that was not directly relevant to conceptual understanding.

Nevertheless, the findings also indicate that extraneous load can be reduced through visual scaffolding, as evidenced by the 'Reduced via Visual Scaffolding' node (f = 5, 3.5%). While small in frequency, this category is pedagogically significant: students who generated their own sketches or received visual guidance showed markedly different cognitive profiles in the radar visualization, with reduced extraneous load indicators and increased engagement in germane load categories such as 'Conceptual Transfer' (f = 14). Participant S76 noted: "Once I drew the semicircle and marked the quarter area, the problem became easy. I could focus on calculating instead of figuring out what the question meant." This aligns with the principles of visual scaffolding and the modality principle (Szabo et al., 2020; Mayer, 2024; Acar et al., 2025), which suggest that integrated visual and verbal presentation of information supports the optimal allocation of cognitive resources by externalizing problem structure and reducing the need for mental reconstruction.

The third finding concerns germane cognitive load, which reflects productive and deep cognitive processing. The emergence of germane load was evidenced by three coded subthemes identified through NVivo analysis: 'Conceptual Transfer' (f = 14), where students connected integral results to circle area formulas; 'Reflective Verification' (f = 9), where they checked consistency across symbolic and geometric representations; and 'Schema Integration' (f = 5), where they articulated relationships among symbolic, visual, and procedural forms. Although these indicators appeared in only a subset of students (19.5% of all coded instances), they demonstrated a qualitatively different level of cognitive engagement. Participant S24 exemplified conceptual transfer: "When I remembered that the formula $y = \sqrt{4 - x^2}$ is from a circle, I could imagine the shape. Then the integral became like finding a part of the circle's area, not just solving an equation." Participant S52 demonstrated reflective verification: "After integrating, I compared my answer with the area formula $\frac{1}{4}\pi r^2$. It matched, so I knew my reasoning was correct." These patterns of reflective thinking are consistent with constructive learning theory (Leppink, 2017; Risko & Dunn, 2015), indicating that these students not only completed computational procedures but also constructed new conceptual schemas linking symbolic meaning with spatial context.

The value of identifying these distinct load profiles is significantly enhanced through the use of radar visualization (**Figure 1**), which functions not merely as a representational tool but as a powerful analytical instrument in its own right. Beyond simply displaying data, the radar chart, grounded in our NVivo coding, deconstructs aggregate cognitive load into its constituent components, revealing the proportional trade-offs and dominance among the specific sub-dimensions of cognitive load an insight impossible to obtain from conventional Likert-scale questionnaires that might only report that students experienced "high intrinsic load." For instance, the radar's axes are empirically-derived thematic nodes. The prominent extension along the 'Symbolic-Visual Confusion' (f=38) axis visually quantifies the dominant source of intrinsic load, while the 'Visual-Spatial Difficulty' (f=21) axis reveals a related but distinct struggle. Similarly, the shape of the 'Extraneous Load' zone clearly shows that 'Ambiguous Verbal Instruction' (f=26) was a far greater source of unnecessary cognitive burden than 'Technical Language Overload' (f=10). This granularity transforms a unidimensional concept of "load" into a multi-dimensional cognitive fingerprint. Conventional questionnaires, by averaging responses, obscure the fact that the cognitive landscape is not uniform. The radar visualization makes this imbalance starkly visible, demonstrating that for most students, cognitive resources were disproportionately consumed by managing the split-attention effect (high on 'Ambiguous Verbal Instruction') and grappling with element interactivity (high on 'Symbolic-Visual Confusion'). Meanwhile, the compressed area of the 'Germane Load' zone with 'Conceptual Transfer' (f=14) and 'Reflective Verification' (f=9) as its most prominent axes provides a visual measure of how rarely this productive load was achieved. The chart, therefore, does not just present data; it diagnoses a problem, visually arguing that the instructional design failed to redistribute cognitive effort away from extraneous processing and toward germane schema construction a conclusion far more compelling when presented as a holistic visual profile than as a table of means and standard deviations.

The interaction among the three dimensions of cognitive load, as visualized in the radar patterns of **Figure 1**, reveals a dynamic continuum from cognitive burden to cognitive investment. The radar visualization shows that students with high intrinsic and extraneous load profiles (large blue and orange zones) exhibited minimal germane load indicators (small green zone), suggesting that when working memory is occupied by decoding task complexity and inefficient presentations, few resources remain for schema construction. Conversely, students who demonstrated germane load indicators occupied a distinct profile zone in the radar visualization, with relatively reduced intrinsic and extraneous load frequencies. This pattern indicates that with the support of visualization and instructional guidance as evidenced by the 'Reduced via Visual Scaffolding' node some students were able to transform cognitive demands into constructive germane load. This dynamic aligns with the hierarchical model of Cognitive Load Theory (Sweller, 2023), in which instructional design plays a crucial role in transforming 'load' into 'meaningful learning' by reallocating cognitive resources from extraneous processing to germane processing.

Pedagogically, the findings of this study underscore the importance of adaptive instructional design grounded in Cognitive Load Theory. Teachers must balance conceptual challenge with clarity of instruction. Strategies such as dual-coding representation, guided visualization, and worked-example fading (Ott et al., 2018) can help students channel their cognitive effort in more productive directions. In addition, reflective activities such as group discussions and peer explanation have also been shown to more effectively promote the formation of new schemas (Park, 2025).

In the context of digital learning, the integration of interactive media such as GeoGebra or Desmos can strengthen the balance of cognitive load (Bokosmaty et al., 2017; Milenković et al., 2022). Dynamic multimodal representations enable students to observe direct relationships between symbolic equations and geometric forms, thereby reducing extraneous load and reinforcing conceptual schema development (Shrestha & Bhattarai, 2020). This approach aligns with the findings of Hammoudi and Grira (2023), which emphasize that interactive visualization plays a crucial role in facilitating germane processing within STEM learning contexts.

The findings of this study culminate in a formalized conceptual framework the Cognitive Load Redistribution Model which articulates how instructional design can strategically manage the three types of cognitive load. This model is grounded in the empirical patterns observed in our NVivo analysis and radar visualization, and is expressed through three formal propositions:

Proposition 1: Visual Scaffolding as an Extraneous Load Mitigator. The high frequency of 'Ambiguous Verbal Instruction' (f=26) and its subsequent reduction through self-generated sketches ('Reduction via Visual Scaffolding', f=5) demonstrate that extraneous load is a function of representational design, not a fixed task property. Strategic integration of visual scaffolds which externalize spatial relationships and explicitly map symbols onto geometric referents directly mitigates the split-attention effect. By offloading mental imagery construction, visual scaffolding frees working memory capacity from decoding ambiguous language, thereby transforming extraneous burden into a manageable condition for learning.

Proposition 2: The Catalytic Role of Guided Element Interactivity. The dominance of 'Symbolic-Visual Confusion' (f=38) confirms that the inherent complexity (element interactivity) of integral geometry generates high intrinsic load. However, this load should not be eliminated but managed through guided interactivity. Instruction should provide structured pathways such as worked examples that explicitly connect algebraic functions to their geometric graphs allowing learners to navigate element relationships without being overwhelmed. This preserves intellectual challenge while preventing cognitive overload.

Proposition 3: Transformation of Intrinsic Complexity into Germane Engagement through Reflection. The emergence of germane load among students exhibiting 'Conceptual Transfer' (f=14) and 'Reflective Verification' (f=9) was contingent upon metacognitive engagement. This proposition formalizes that transitioning from coping with intrinsic load to investing in germane load requires a pedagogical shift from task completion to conceptual reflection. When instruction prompts students to verify results across multiple representations or articulate procedural rationales, it encourages schema integration. In this model, germane load is not a separate category of effort, but the productive outcome of managing intrinsic load through tasks designed to foster reflection and abstraction.

In essence, this model posits that effective mathematics instruction is not about reducing cognitive load entirely, but rather about strategically redistributing it (Castro-Alonso et al., 2021; Kosch et al., 2023). The inherent complexity of the material (intrinsic load) should be preserved as a source of intellectual challenge, whereas inhibitory factors (extraneous load) must be minimized. Through this redistribution, students' cognitive resources can be directed toward constructive processing (germane load). This transformation from "load" to "cognitive investment" represents the essence of meaningful learning in the context of visual and symbolic mathematics education.

### Limitations and future research

While this study provides an in-depth qualitative mapping of cognitive load profiles, its findings must be interpreted within the context of certain limitations. First, the data collection was confined to a single public university located in eastern Indonesia. This geographical and environmental context may limit the generalizability of the identified cognitive profiles, as students' prior educational experiences, cultural backgrounds, and familiarity with specific instructional methods can influence how they manage cognitive load. Future research should address this limitation by conducting comparative studies across diverse universities in different regions, both within Indonesia and internationally. Such multi-site investigations would help validate the robustness of the cognitive profiles identified here and determine whether the observed dominance of symbolic-visual confusion and the mitigating role of visual scaffolding are universal phenomena or are moderated by specific educational contexts. Expanding the sample to include a wider range of institutional types and student populations would significantly enhance the external validity of the proposed Cognitive Load Redistribution Model. Future studies may also examine how different instructional designs influence the development of students' mathematical reasoning while regulating intrinsic, extraneous, and germane cognitive load in visually complex mathematical tasks.

## CONCLUSION

This study aimed to map students' cognitive load profiles in solving integral geometry problems by integrating NVivo-assisted thematic analysis with radar visualization. Based on the analysis of the thinking processes of 144 university students, the findings show that the three main components of Cognitive Load Theory intrinsic, extraneous, and germane load emerge at varying intensities and form diverse cognitive patterns across individuals.

First, intrinsic cognitive load was identified as the most dominant type of load, evidenced by the high frequency of 'Symbolic-Visual Confusion' (f = 38, 26.4%) and 'Visual-Spatial Difficulty' (f = 21, 14.6%) in the NVivo coding. This dominance was primarily driven by high element interactivity when students attempted to connect the symbolic representation of the function $y = \sqrt{4 - x^2}$ with the geometric form of a semicircle, interpret the integral boundaries, and visualize the quarter-circle region, as reflected in verbal indicators such as 'cannot relate function to circle' and 'confused about which area.' Difficulties in coordinating symbolic, visual, and spatial information overloaded students' working memory, especially among those who lacked well-developed conceptual schemas relating geometry and integral calculus.

Second, the study found that extraneous cognitive load remained relatively high, as shown by the 'Ambiguous Verbal Instruction' node (f = 26, 18.1%) and confirmed by students' verbal reports of confusion when textual and visual information were separated. Coded indicators such as 'text without picture confused me' (n = 18) and visual errors including 'misplacing the quarter-region' (f = 12) demonstrated that ambiguous instructions, unfamiliar technical terminology, and the absence of initial visual scaffolding caused students to allocate cognitive resources to interpreting the task context rather than understanding the underlying concepts. This indicates that suboptimal instructional design can unnecessarily increase cognitive burden. Nevertheless, the use of visual scaffolds either provided or self-generated by students proved effective in reducing this avoidable load.

Third, germane cognitive load emerged among a smaller subset of students (19.5% of coded instances) who demonstrated 'Conceptual Transfer' (f = 14), 'Reflective Verification' (f = 9), and 'Schema Integration' (f = 5) in the NVivo thematic analysis. These students not only completed integral procedures but also constructed conceptual understanding of the relationship between the area of a circle and its integral model, as illustrated by their ability to connect symbolic computations with geometric reasoning and verify results across multiple representations. The presence of germane load reflects productive cognitive engagement, where students not only completed the integral procedures but also constructed a conceptual understanding of the relationship between the area of a circle and its integral model. Although less frequent, this pattern highlights the potential for deeper schema construction through appropriate instructional support. Importantly, these patterns also suggest that effective management of germane cognitive load can facilitate the emergence of mathematical reasoning, as students integrate symbolic expressions, geometric representations, and conceptual interpretations during the problem-solving process.

Overall, the findings indicate that students' cognitive load profiles in solving integral geometry problems are imbalanced, as visualized in the radar patterns of **Figure 1** where intrinsic and extraneous load zones (blue and orange) dominate and often constrain the development of germane load (green zone). These results imply that to optimize integral geometry learning, instructional design must intentionally reduce extraneous load, manage the complexity of intrinsic load, and promote germane load through the use of visual representations, multimodal scaffolding, and reflective activities. In doing so, unproductive cognitive burden can be transformed into meaningful conceptual processing, supporting more effective, adaptive, and deep mathematical learning. Through such instructional design, cognitive effort can be redirected from merely managing task complexity toward supporting deeper conceptual understanding and mathematical reasoning in visually intensive mathematical contexts.

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**Ethical statement**

Ethical approval was obtained from Universitas Muhammadiyah Makassar. Written informed consent was obtained from teachers, and verbal assent was obtained from students after explaining the purpose of the study, its voluntary nature, and their right to withdraw at any time without consequence.

**Competing interests**

The authors declared no competing interests.

**Author contributions**

All authors jointly contributed to the conception and design of the study, data collection and analysis, manuscript preparation, critical revision, and final approval. All authors reviewed and approved the final version of the manuscript and agree to be accountable for all aspects of the work.

**Data availability**

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

**AI disclosure**

The authors used AI-assisted tools (Grammarly and DeepSeek) solely for language editing and improving the clarity of the manuscript. No AI tools were used in data analysis, interpretation, or decision-making. All authors take full responsibility for the content of this article and its scientific integrity, in accordance with COPE guidelines.

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