

Research paper

Pre-Service Mathematics Teachers' Intra-Mathematical Connections: An Onto-semiotic Analysis of Explanations of Exponential and Logarithmic Functions

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ABSTRACT

This study analyzed the intra-mathematical connections activated by pre-service mathematics teachers when explaining exponential and logarithmic functions in a Colombian university context. The research was theoretically grounded in the Extended Theory of Connections and the Onto-semiotic perspective, which allow the analysis of relationships established between mathematical concepts, procedures, meanings, and representations during instructional processes. Using an exploratory qualitative comparative case-study design involving two purposively selected pre-service mathematics teachers, the study was conducted in four stages: lesson design, case selection, participant observation in a Didactics of Calculus course, and thematic and onto-semiotic analysis through the networking between ETC and OSA. The findings revealed an interconnected network of mathematical connections, including instruction-oriented, meaning, feature, part-whole, procedural, implication, reversibility, and representational connections. These connections showed how the PMTs coordinated mathematical objects, processes, and representations when explaining exponential and logarithmic functions. Particularly, algebraic, graphical, tabular, and verbal representations supported the construction of inverse relationships. Reversibility emerged as the most structurally complex connection and was manifested through recognition, procedural, and structural forms. Rather than confirming a meta-connection, the findings suggest that reversibility may function as an integrative connection, coordinating meanings, procedures, properties, representations, and logical reasoning. The study concludes that the analysis of mathematical connections provides a valuable framework for strengthening didactic-mathematical knowledge in initial teacher education and understanding how pre-service teachers communicate mathematical meanings during teaching processes.

Keywords: intra-mathematical connections, exponential and logarithmic functions, pre-service mathematics teachers, mathematics education

Mathematical connections are a well-established field in mathematics education due to their role in strengthening conceptual understanding and linking mathematics with everyday life (Mhlolo, 2012; Rodríguez-Nieto et al., 2021a, 2021b, 2024). Research across educational levels has explored connections related to exponential, logarithmic, and derivative concepts, revealing persistent difficulties in coordinating symbolic, graphical, and verbal representations (Campo-Meneses & García-García, 2020; García-García & Dolores-Flores, 2018, 2021; Pino-Fan et al., 2015, 2018; Rodríguez-Nieto et al., 2021a, 2021b, 2022a). These findings highlight the need to further investigate how students establish mathematical connections and how their absence affects problem-solving.

In this context, the teacher plays a fundamental role as a mediator of learning, capable of implementing strategies that promote understanding and student engagement (Realpe Vidal, 2022). A key strategy is fostering mathematical connections, as these enable students to construct meaning, relate concepts, and apply knowledge across contexts through tasks that integrate models, procedures, meanings, properties, and representations (Eli et al., 2013; Anugraheni et al., 2025; National Council of Teachers of Mathematics [NCTM], 2000; Rodríguez-Nieto & Font, 2025; Sudirman et al., 2026).

Developing meaningful understanding is particularly relevant in pre-service teacher education, where conceptual and applied knowledge must be integrated (Rodríguez-Nieto et al., 2025). Although contextualized problems can support richer interpretations of exponential and logarithmic functions (Borji et al., 2025), studies report persistent difficulties, including limited algorithmic mastery and weak conceptual understanding, especially in real-world contexts. These challenges are associated with gaps in prior knowledge and difficulties in applying logarithmic laws (Okoye-Ogbalu & Nnadozie, 2024), despite evidence that well-designed instructional activities can promote deeper understanding.

Exponential and logarithmic functions are closely related, as one is the inverse of the other, which requires teachers not only to understand their formal structure but also to communicate this relationship effectively (Díaz-Berrios & Martínez-Planell, 2022). As Rodríguez et al. (2021) note, “the logarithmic operator process is constructed in parallel with the exponential relation process through the invariant property of the exponential, thus facilitating the parallel emergence of the exponential and logarithmic functions” (p. 162). Consequently, the ability to identify and communicate these relationships is essential for supporting students’ understanding; otherwise, instruction may become fragmented and hinder the construction of meaning.

Students often experience significant difficulties in understanding and representing logarithmic functions, partly because instruction is frequently restricted to symbolic and theoretical aspects (Mulqueeny, 2012). In this sense, mathematical connections can serve as a key resource for overcoming these difficulties, as they enable links between symbolic, graphical, procedural, and structural aspects of functions. Businkas (2008) argues that “students and teachers use mathematical connections of various types, such as procedural, different representations, feature, part-whole, etc., when solving mathematical tasks and in the teaching and learning process” (p. 3). In this perspective, connections are understood as meaningful relationships between concepts, ideas, procedures, or representations that allow students to relate mathematics to real-life situations (García-García & Dolores-Flores, 2018).

Furthermore, mathematical connections contribute to the development of reasoning and communication (Bingölbalı & Coşkun, 2016; Campo-Meneses & García-García, 2020). Mhlolo (2012) emphasizes their importance in the development of mathematical understanding, as they support logical thinking, argumentation, and analysis. In line with this, the NCTM (2000) highlights the importance of learning mathematics with understanding, promoting reasoning, imagination, and knowledge construction based on prior knowledge, as well as its transfer to other contexts.

The study of exponential and logarithmic functions is particularly relevant due to their conceptual importance and applicability in modeling real-world phenomena (Campo-Meneses & García-García, 2020). Understanding these functions enables students to engage with more advanced mathematical processes and solve diverse contextualized problems. However, teaching practices often remain limited to linear or procedural approaches, which restrict the recognition of the logarithmic–exponential structure of phenomena (Sureda & Otero, 2013). Previous research shows that many university students fail to establish connections between exponential and logarithmic laws or between their graphical, symbolic, and verbal representations (Campo-Meneses & García-García, 2020).

In addition, there is a tendency to focus primarily on the natural logarithm, which limits the understanding of its general applicability in algebraic contexts (Escobar, 2014) and reinforces a mechanical and disconnected use of algorithms (Campo-Meneses & García-García, 2021). In this sense, establishing mathematical connections particularly reversibility becomes essential for understanding the relationship between exponential and logarithmic functions (Campo-Meneses & García-García, 2020, 2021).

Although research on exponential and logarithmic functions has made significant progress, persistent challenges remain. Studies highlight the potential of modeling and real-world connections (Araújo et al., 2022; Loi

et al., 2020), as well as the role of didactic and technological approaches (Koştur & Yılmaz, 2017; Porras, 2025). At the same time, other research underscores difficulties in handling logarithmic rules (Okoye-Ogbalu & Nnadozie, 2024; Abdurahmonov, 2025). Overall, the literature reveals advances, but also the need to strengthen more structural forms of connection.

Recent research on exponential and logarithmic functions demonstrates a growing interest in moving beyond instruction focused solely on algebraic rules and fostering a conceptual, representational, and applied understanding. The mental construction of the exponential function through repeated multiplication has been studied, highlighting the need to articulate mathematical processes and objects (Díaz-Berrios & Martínez-Planell, 2025), while other research shows the potential of digital technologies such as TikTok and interactive materials to promote meaningful learning and improve the understanding of these functions (Porras, 2025; Vicuña Espinoza, 2026). Likewise, mathematical laboratories and didactic analysis encourage experimentation, visualization, and coordination among algebraic, graphical, and numerical representations (Codogni, 2025; Faz & Navas, 2026). Modeling phenomena, such as the bounce of a ball, also contributes to interpreting exponential behavior in contextualized situations (Silva, 2026). However, difficulties persist in understanding the inverse exponential-logarithmic relationship, coordinating representations, justifying procedures, and connecting properties. Therefore, research is needed to examine how students and teachers articulate meanings, representations, procedures, and reversibility to build a structural and connected understanding of both functions.

While previous studies have analyzed mathematical connections among students and in-service teachers (Campo-Meneses & García-García, 2023, 2025), as well as their presence in curricula (Campo-Meneses et al., 2023), less attention has been given to how these connections are activated in initial teacher education (Codogni, 2025; Faz & Navas, 2026; Silva, 2026). In particular, the specific contribution of this study lies in examining how pre-service mathematics teachers coordinate these connections while explaining exponential and logarithmic functions, using an integrated ETC–OSA analysis. Therefore, this study aims to analyze the mathematical connections activated by pre-service mathematics teachers when explaining exponential and logarithmic functions in a calculus didactics classroom.

THEORETICAL FRAMEWORK

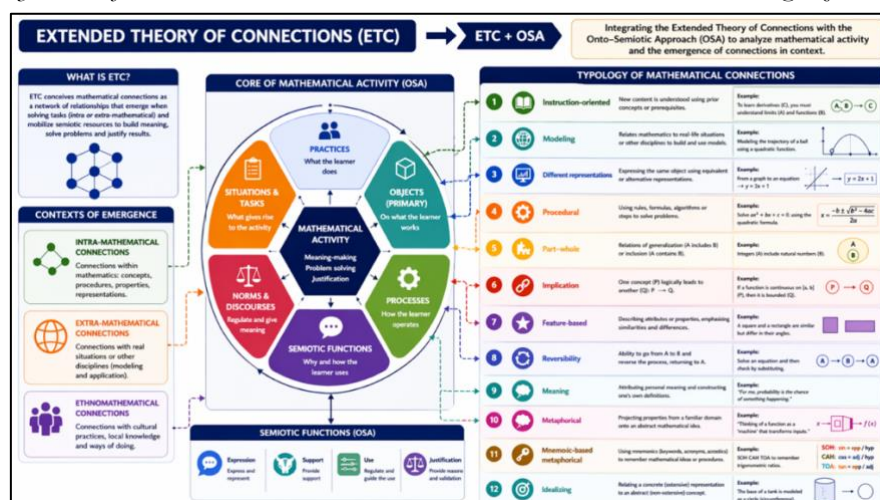
Extended theory of connections

The Extended Theory of Connections (ETC) provides a framework for identifying and characterizing the relationships established among mathematical concepts, procedures, properties, representations, and reasoning during mathematical activity. Rather than considering connections as isolated associations, the ETC organizes them into a typology that allows their manifestations in mathematical practices to be systematically analyzed.

In this study, a mathematical connection is understood from an onto-semiotic perspective as a relational construct emerging from a network of practices, processes, objects, and semiotic functions (Rodríguez-Nieto et al., 2022a). In this sense, connections are not isolated elements but part of a broader system of mathematical activity. They can be classified as intra-mathematical, established between mathematical concepts, procedures, theorems, arguments, and representations (Dolores-Flores & García-García, 2017), and extra-mathematical, which establish relationships between mathematical concepts or models and problems in non-mathematical contexts, or vice versa (Dolores-Flores & García-García, 2017). **Figure 1** presents a synthesis of mathematical connections.

Figure 1

Synthesis of mathematical connections with an onto-semiotic view based on Rodríguez-Nieto et al. (2024)



Source: Authors' own elaboration.

Mathematical connections play a central role in conceptual understanding, as they enable the articulation of meanings within mathematical activity (Rodríguez-Nieto et al., 2025). Among the different types of connections, several are particularly relevant in this study. Instructional connections involve linking new concepts to prior knowledge, supporting the construction of meaning (Businkas, 2008; Mhlolo, 2012). Procedural connections refer to the application of rules, algorithms, and formulas, reflecting operational knowledge (García-García & Dolores-Flores, 2021). In contrast, part-whole connections establish relationships of inclusion or generalization, where a mathematical object is understood as part of a broader structure (Businkas, 2008; García-García & Dolores-Flores, 2021).

Additionally, implication connections are based on logical “if-then” reasoning, allowing conclusions to be derived from given premises (Mhlolo, 2012; García-García & Dolores-Flores, 2021). Connections between representations arise when mathematical objects are expressed across multiple semiotic registers (such as algebraic, graphical, or verbal) facilitating different forms of interpretation (Businkas, 2008).

The feature connection focuses on identifying properties and characteristics that define or differentiate mathematical objects (Eli et al., 2011), while the meaning connection involves attributing conceptual understanding, including definitions and contextual interpretations (García-García & Dolores-Flores, 2021).

Of particular relevance is the reversibility connection, which involves bidirectional transformations between mathematical objects, enabling movement from one representation or process to another and back again (García-García & Dolores-Flores, 2021). This type of connection reflects a deeper understanding of structural relationships. Complementarily, metaphorical connections support the interpretation of abstract concepts through familiar experiences, while idealization connections involve transitioning from concrete representations to abstract mathematical models (Rodríguez-Nieto et al., 2022b, 2024; Ledezma et al., 2024).

Together, these connections illustrate the relational nature of mathematical thinking and provide a framework for analyzing mathematical activity in educational contexts.

In this study, the expression “integrative connection” is used as an analytical descriptor rather than as a new category within the ETC. Reversibility is interpreted as integrative when, within the same mathematical episode, a bidirectional relationship between exponential and logarithmic functions is supported through the explicit coordination of at least two additional connections, such as meaning, procedural, representational, feature, part-whole, or implication connections. Their mere co-occurrence is not considered sufficient; the participant must establish a relationship showing how these connections contribute to explaining, representing, justifying, or validating the inverse correspondence. Given the exploratory nature of the study, this interpretation is restricted to the two cases analyzed and is not proposed as a new theoretical classification or as a general property of reversibility.

Onto-semiotic approach (OSA)

The Onto-Semiotic Approach (OSA) provides a comprehensive framework for analyzing mathematical knowledge and instructional processes. It conceptualizes mathematical activity as a system of practices in which subjects solve problems, communicate, justify, and generalize results (Godino & Batanero, 1994).

Within these practices, configurations of primary objects such as problems, language, definitions, procedures, and arguments are dynamically articulated (Godino et al., 2007; Font et al., 2013). These configurations can be analyzed through different dualities (e.g., institutional/personal or expression/content), highlighting the complexity of mathematical knowledge. A central concept in OSA is the semiotic function, defined as a triadic relationship between an expression and its content, mediated by a subject (Font, 2007). From this perspective, mathematical knowledge is understood as a network of interrelated objects and practices, allowing mathematical connections to be interpreted as specific instances of semiotic functions (Rodríguez-Nieto et al., 2022a).

Networking between extended theory of connections & onto-semiotic approach

The networking of theories makes it possible to articulate different theoretical frameworks to gain a deeper understanding of the complexity of mathematics teaching and learning processes while respecting their conceptual and methodological principles (Kidron & Bikner-Ahsbabs, 2015; Prediger et al., 2008). This research adopts the networking between the ETC and the OSA developed by Rodríguez-Nieto et al. (2022a). Although both frameworks use content analysis, they differ methodologically: the ETC identifies and classifies mathematical connections according to a predefined theoretical typology, whereas the OSA provides tools for examining mathematical activity through practices, primary objects (PO), processes, and semiotic functions (SF). Following the networking stages proposed by Drijvers et al. (2013), Kidron and Bikner-Ahsbabs (2015), and Radford (2008), students’ written and verbal productions can therefore be analyzed from both perspectives.

The two frameworks are complementary because they operate at different but interconnected analytical levels. The ETC enables the identification and classification of mathematical connections, while the OSA reveals their

underlying structure and functioning by reconstructing the practices, primary objects, processes, and semiotic functions involved. Thus, mathematical connections can first be identified and classified through the ETC and subsequently analyzed using OSA tools to reconstruct the mathematical practices, primary objects, processes, and semiotic functions underlying them. Metaphorically, a mathematical connection identified through the ETC represents the tip of an iceberg, whereas the OSA makes visible the deeper network of mathematical practices, objects, processes, and semiotic functions that supports it. This networking provides a more comprehensive analytical framework for examining the mathematical connections emerging in the PMTs' mathematical activity and productions.

METHODOLOGY

This study adopted an exploratory qualitative comparative case-study design involving two cases (PMT1 and PMT2). This design was selected to provide an in-depth examination of how mathematical connections were activated, coordinated, and communicated within a specific instructional setting, rather than to seek statistical generalization to the broader population of pre-service mathematics teachers. The comparative perspective supported both within-case and cross-case analyses, enabling a detailed examination of each participant's mathematical activity as well as the identification of convergences and differences between the cases. The study was organized into four stages: (i) design of a lesson on exponential and logarithmic functions, (ii) purposive selection of the cases, (iii) participant observation during lesson implementation, and (iv) thematic and onto-semiotic analysis through the networking between ETC and OSA.

The cases were bounded by the instructional setting, mathematical content, and period of observation. Specifically, the investigation focused on the planning and classroom explanation of exponential and logarithmic functions within a Didactics of Calculus course, including definitions, properties, procedures, representations, and the inverse relationship between both functions. The temporal boundary comprised the observed and video-recorded instructional sessions in which the lesson was implemented. Accordingly, the unit of analysis was the mathematical activity evidenced during these sessions, operationalized through the practices and connections manifested in verbal discourse, written productions, procedures, representations, and arguments. This delimitation allowed the analysis to characterize and compare not only the types of connections identified through ETC, but also their underlying onto-semiotic configurations and levels of complexity through OSA.

Participants and context

The participants were purposively selected as information-rich cases rather than as a representative sample of the broader population of pre-service mathematics teachers. The selection criteria included high academic performance (average grade of 4.5/5), active participation in the program, advanced curricular progress, and demonstrated mathematical argumentation. Selecting these cases was theoretically and analytically relevant because it provided favorable conditions for examining the diversity and complexity of mathematical connections that may emerge when PMTs with strong mathematical performance explain exponential and logarithmic functions. Therefore, the selection was intended to support an in-depth examination of the phenomenon rather than statistical representativeness or population-level generalization. Given the bounded comparative case-study design and the purposive selection of two information-rich cases, data saturation was not used as a criterion for determining the number of participants.

Data Collection

Data were collected through participant observation of the teaching sessions in which PMT1 and PMT2 explained exponential and logarithmic functions. The sessions were video-recorded in their entirety, allowing the researchers to capture the participants' verbal discourse, written productions, gestures, use of representations, and interactions with mathematical resources. Field notes and photographic records were used as complementary sources. During the lessons, the participants used audiovisual and physical resources, including a projector, whiteboard, and ruler, to support symbolic, numerical, tabular, graphical, and verbal representations of the mathematical concepts.

The researcher adopted a participant-observer role with limited intervention. The PMTs were initially allowed to develop their explanations autonomously, so that their mathematical discourse and connections could emerge spontaneously. Researcher intervention occurred only when clarification or further elaboration of an idea already introduced by a participant was considered necessary. The prompts were open-ended and non-directive and were intended to encourage participants to explain, justify, or relate ideas already present in their discourse, rather than introduce new mathematical content or suggest a particular type of connection. Examples of prompts included questions such as *"Could you explain this idea further?"*, *"Why does this happen?"*, *"How is this representation related to the previous one?"*, or *"What relationship do you observe between these two functions?"*

Data analysis

The data were analyzed using thematic analysis, which enables the identification, organization, and interpretation of recurring patterns in qualitative data. This approach is particularly valuable in educational research, as it facilitates the understanding of complex phenomena by capturing participants' experiences and meaning-making processes. Thematic analysis provides both flexibility and analytical rigor, allowing for inductive or deductive examination depending on the theoretical framework adopted. Following Braun and Clarke (2006), the analysis was conducted through a series of recursive phases.

Phase 1: Familiarization with the data

The information was obtained through participant observation of classes taught by the PMT's, which were recorded, transcribed, and reviewed by the researchers. In this phase, the transcripts were repeatedly read, allowing for a deeper understanding of the participants' mathematical discourse. Relevant fragments of the teaching sessions were also analyzed and compared with field notes and supporting images, facilitating initial immersion in the data.

Phase 2: Generation of initial codes

Expressions and discursive fragments were identified that demonstrated the use of different mathematical connections during the explanation of the functions under study. For example:

The phrase “the logarithmic function is the inverse of the exponential function” is a code associated with the connection of reversibility.

The phrase “the logarithm finds the exponent to which the base was raised” is a code that reveals the connection of meaning.

The phrase “we represent the function with a table of values and then graph it” is a code that demonstrates the connection of different representations.

These examples of codes were assigned both to individual fragments and to sequences of didactic actions, reflecting recurring practices in the teaching process. In this context, the coding was carried out systematically and iteratively, segmenting the transcripts into units of meaning. Each fragment was analyzed considering: (i) the mathematical object involved, (ii) the action performed, and (iii) the relationship established between objects.

The codes were constructed as empirical indicators of relationships between mathematical objects in discourse. These relationships were first classified according to the ETC and subsequently analyzed as semiotic functions using OSA tools. The codes were derived directly from excerpts of the transcription:

C1: “The exponential function is...” → formal definition of the exponential function.

C2: “The logarithmic function is...” → formal definition of the logarithmic function.

C3: “The logarithm seeks the exponent...” → meaning of the logarithm.

C4: “a is the base...” → structural elements or properties.

C5: “The domain goes...” → domain of the function.

C6: “It is increasing...” → behavior of the function's graph.

C7: “We apply the property...” → algebraic rules.

C8: “We replace x...” → substitution.

C9: “We construct the table...” → table of values.

C10: “We graph...” → graphical representation.

C11: “This corresponds...” → translation between representations.

C12: “The asymptote...” → asymptotes.

C13: “We take these points...” → key points.

C14: “Let us recall...” → prior knowledge.

C15: “First..., then...” → sequencing.

C16: “As we saw...” → recap.

C17: “They are inverses...” → recognition of inverse.

C18: “We interchange x and y” → algebraic inversion.

C19: “We solve for...” → inverse procedure.

C20: “If $x = 3$ then...” → inverted ordered pairs.

C21: “The domain now...” → domain–range inversion.

C22: “The asymptote changes...” → structural comparison.

C23: “If..., then...” → implication.

The co-occurrence of codes was allowed, showing that a single practice can activate multiple mathematical connections.

Phase 3: Search for themes and subthemes

The construction of subthemes and themes was carried out through a process of progressive abstraction, constant comparison, and hierarchical organization, in which the codes generated in the previous phase were grouped into higher-level units of meaning. This process was interpretive rather than merely classificatory, aiming to identify patterns in how PMTs structure and communicate mathematical knowledge.

Initially, codes were analyzed according to their semantic and functional affinity, grouping those that reflected similar mathematical relationships or discursive practices. This allowed the transition from micro units (codes) to meso units (subthemes), understood as specific ways in which mathematical connections are manifested (Rodríguez-Nieto & Font, 2025), characterized by recurring actions such as defining, calculating, representing, justifying, or comparing.

For example, codes C1 and C2 (formal definitions) and C3 (logarithmation as a process) were grouped into subtopics related to meaning construction, since they all involve attributing meaning to mathematical objects. Similarly, codes C18 (variable exchange) and C19 (algebraic solving) were grouped into a subtopic describing the algebraic process of inversion, which is subsequently integrated into the reversibility connection. At a second level of abstraction, the subtopics were integrated into themes, understood as broader configurations corresponding to the typologies of mathematical connections proposed in the ETC. Each theme represents a relational structure between mathematical objects, mediated by semiotic practices and functions.

A key aspect of this phase was the identification of different analytical forms within each topic, which allowed us to go beyond the simple presence of a connection and analyze its degree of development. For example, in the reversibility connection, three clearly differentiated levels were identified: A recognition level, where the PMT states that the logarithmic function is the inverse of the exponential function (C17), without further conceptual development. A procedural level, where operations such as variable interchange and algebraic rearrangement are performed (C18, C19), demonstrating an operational understanding of inversion. A structural level, where correspondences between ordered pairs are established (C20), the inversion between domain and range is recognized (C21), and graphical properties such as asymptotes are compared (C22), indicating a deeper understanding of the functional relationship.

Furthermore, cross-cutting relationships between themes were identified, demonstrating that mathematical connections do not operate in isolation. For example, the reversibility connection depends on the use of algebraic procedures (Procedural Theme) and on the coordination between representations (Different Representations Theme). Similarly, the meaning connection is linked to the instructional connection when teachers use prior knowledge to construct definitions.

Each theme can be interpreted as a configuration of semiotic functions, in which different mathematical objects (concepts, procedures, representations) are related through discourse. For example, in the connection of different representations, semiotic functions are identified that link algebraic expressions, tables of values, and graphs, while in reversibility, relationships are established between two functions through inverse transformations. This phase allowed for the construction of a solid hierarchical analytical structure, in which codes represent specific empirical evidence, subthemes represent patterns of mathematical action, and themes represent conceptual structures of mathematical connection.

This organization not only provides the basis for the table presented in Phase 4 but also allows for a rigorous definition of the topics in Phase 5 and a deep interpretation of the study results.

Theme 1: Connection of meaning

This theme encompasses the practices in which PMTs develop a conceptual understanding of exponential and logarithmic functions. It is structured around the following subthemes:

Subtheme 1.1: Functional Definition $\rightarrow$ (C1, C2)

Subtheme 1.2: Logarithms as an Inverse Process $\rightarrow$ (C3)

Subtheme 1.3: Structural Elements $\rightarrow$ (C4)

In this theme, meaning is constructed through the articulation of formal definitions, explanations of the logarithmic process, and the identification of components such as base, exponent, and argument. From the perspective of the OSA, semiotic functions are evident that relate symbolic expressions to conceptual meanings. However, the analysis shows that these meanings remain largely within the realm of mathematics, with little connection to applied contexts.

Theme 2: Feature connection

This theme covers the practical exercises in which PMTs identify and describe structural properties of functions. It consists of:

Subtheme 2.1: Domain and Range $\rightarrow$ (C5)

Subtheme 2.2: Functional Behavior $\rightarrow$ (C6)

Subtheme 2.3: Algebraic Properties $\rightarrow$ (C7)

This demonstrates an understanding based on the attributes of functions, such as growth, constraints, and overall behavior. However, these characteristics are treated primarily descriptively, without being integrated into explanations that allow for inferring properties or generalizing behaviors.

Theme 3: Procedural connections

This theme focuses on the use of algorithms, rules, and operational sequences to solve mathematical problems. It consists of:

Subtheme 3.1: Substitution of values $\rightarrow$ (C8)

Subtheme 3.2: Construction of tables $\rightarrow$ (C9)

Subtheme 3.3: Application of algebraic properties $\rightarrow$ (C7)

The analysis shows a predominance of instrumental practices, in which procedures are executed correctly, but with little conceptual justification. From the OSA perspective, a weak articulation between procedures and meanings is observed, which limits deep understanding.

Theme 4: Part-whole connection

This theme groups practices in which participants relate partial elements of a function to its overall behavior. It consists of:

Subtheme 4.1: Identifying Asymptotes $\rightarrow$ (C12)

Subtheme 4.2: Significant Points $\rightarrow$ (C13)

Subtheme 4.3: Graphical Relationship–Expression $\rightarrow$ (C10, C13)

A structured organization of the graphical construction is evident; however, the relationship between the parts and the whole remains at an operational level, without consolidating into a comprehensive functional understanding.

Theme 5: Instruction-oriented connection

This theme reflects the didactic dimension of the discourse, in which future teachers organize the content based on prior knowledge. It includes:

Subtheme 5.1: Activating prerequisites $\rightarrow$ (C14)

Subtheme 5.2: Didactic sequencing $\rightarrow$ (C15)

Subtheme 5.3: Recapitulation $\rightarrow$ (C16)

A clear pedagogical intention is evident; however, the connection between prior and new knowledge is not always explored in depth, which limits the construction of more complex conceptual networks.

Theme 6: Connecting different representations

This theme groups practices that utilize multiple semiotic registers. It consists of:

Subtheme 6.1: Symbolic representation $\rightarrow$ (C1, C2)

Subtheme 6.2: Graphic representation $\rightarrow$ (C10)

Subtheme 6.3: Coordination of representations $\rightarrow$ (C11, C9, C10)

Although participants employ different registers, the articulation between them is limited by the explanation provided.

Theme 7: Reversibility connection

This is one of the most complex and relevant themes, as it involves a bidirectional understanding of functions. It comprises:

Subtheme 7.1: Recognizing the Inverse $\rightarrow$ (C17)

Subtheme 7.2: Algebraic Process of Inversion $\rightarrow$ (C18, C19)

Subtheme 7.3: Correspondence of Ordered Pairs $\rightarrow$ (C20)

Subtheme 7.4: Domain-Range Inversion $\rightarrow$ (C21)

Subtheme 7.5: Structural Comparison (Asymptotes) $\rightarrow$ (C22)

The analysis allowed us to identify levels of development: from a superficial recognition to a structural understanding that articulates the algebraic, numerical, and graphical aspects. This theme demonstrates how participants begin to construct a systemic view of functions, although still with limitations in their formalization.

Theme 8: Implication connection

This theme addresses the use of logical reasoning in mathematical discourse:

Subtheme 8.1: Logical-deductive reasoning $\rightarrow$ (C23)

Although “if...then...” structures are identified, they remain largely implicit and are not developed as formal arguments, which limits the potential of this connection to strengthen mathematical reasoning. This thematic breakdown not only organizes teaching practices according to their patterns of mathematical connection but also allows us to observe how multiple dimensions of didactic-mathematical knowledge are integrated into teaching. Furthermore, it reveals the potential of thematic analysis to connect the empirical with the theoretical aspects of Mathematical Theory.

Phase 4: Review of the themes

Coding was conducted collaboratively by the research team. The themes and subthemes were reviewed through an investigator triangulation process. Three researchers specializing in mathematical connections reviewed the complete coding matrix (C1–C23) before discussing their observations, which allowed the coding criteria to be refined and the internal consistency of each theme to be examined. Differences in interpretation were discussed collectively and resolved through consensus. For example, uses of the term “meaning” were clearly distinguished from those that corresponded to the “feature” type of connection. The finalized coding structure comprised eight themes and the complete set of codes C1–C23. The metaphorical episode subsequently discussed in the Results section was not included in this formal coding and triangulation process and is therefore presented only as an emergent exploratory observation. [Table 1](#) shows the organization of the themes.

Table 1

Operationalization of codes, and creation of themes through ETC-OSA networking

Codes	Subtheme / empirical indicator	ETC connection	OSA primary objects (PO)	OSA mathematical processes	Semiotic function (Expression → Content)
C1, C2	Formal definition	Meaning	Language, concepts/definitions, propositions	Communication; definition; representation–meaning	Symbolic/verbal definition → meaning of exponential/logarithmic function
C3	Logarithmation	Meaning	Language, concept, procedure	Communication; definition; procedures; representation–meaning	“The logarithm seeks the exponent” → conceptual meaning of logarithmation
C4	Structural elements	Meaning	Language, concepts, propositions	Communication; definition; analysis; particularization	Base/exponent/argument → structural components of the function
C5	Domain and range	Feature	Concepts, propositions/properties, language	Enunciation; analysis; generalization–particularization	Function/condition → domain and range properties
C6	Behavior	Feature	Graphical language, concepts, propositions	Communication; enunciation; analysis; generalization	Graph/table → increasing/decreasing behavior
C7	Algebraic properties	Feature / Procedural	Propositions/properties, procedures, language	Enunciation; procedures; argumentation; generalization–particularization	Algebraic property → valid transformation/procedure
C8	Substitution	Procedural	Procedures, language, concepts	Procedures (algorithms); particularization	General expression + selected value → particular numerical result
C9	Table of values	Procedural / Representations	Procedures, tabular language, concepts	Procedures; materialization–representation; particularization; synthesis	Algebraic expression → numerical values → table
C10	Graph	Representations / Part–whole	Graphical language, procedures, concepts	Procedures; materialization–representation; synthesis	Points/table/algebraic expression → graph
C11	Translation between representations	Representations	Symbolic, numerical, tabular and graphical language	Communication; materialization–representation–meaning; synthesis–analysis	Representation A ↔ representation B of the same mathematical object
C12	Asymptotes	Part–whole	Concepts, propositions, graphical language	Enunciation; analysis; particularization	Asymptote → structural component/property of the whole graph
C13	Key points	Part–whole	Ordered pairs, graphical language, procedures	Procedures; particularization; synthesis; materialization–representation	Selected points → components used to construct the global graph

Codes	Subtheme / empirical indicator	ETC connection	OSA primary objects (PO)	OSA mathematical processes	Semiotic function (Expression $\rightarrow$ Content)
C14	Prerequisites	Instruction	Concepts, propositions, procedures	Communication; personalization– institutionalization; meaning	Prior mathematical knowledge $\rightarrow$ meaning of the new object
Codes	Subtheme / empirical indicator	ETC connection	OSA primary objects (PO)	OSA mathematical processes	Semiotic function (Expression $\rightarrow$ Content)
C15	Sequencing	Instruction	Practices, procedures, concepts	Communication; procedures; synthesis– analysis; institutionalization	Ordered mathematical actions $\rightarrow$ organized instructional sequence
C16	Recap	Instruction	Concepts, propositions, arguments	Communication; synthesis; personalization– institutionalization	Previously established knowledge $\rightarrow$ current mathematical explanation
C17	Recognition of inverse	Reversibility	Concepts/functions, propositions, language	Enunciation; analysis; representation– meaning	Exponential function $\leftrightarrow$ logarithmic function as inverse objects
C18, C19	Algebraic inversion	Reversibility	Symbolic language, procedures, concepts	Procedures (algorithms); analysis– synthesis; materialization– representation	$y = a^x \leftrightarrow x = \log_a y$
C20	Inverted ordered pairs	Reversibility	Ordered pairs, numerical/tabular language	Procedures; materialization– representation; analysis; generalization– particularization	$(x, y) \leftrightarrow (y, x)$
C21	Inverse domain–range	Reversibility	Concepts, propositions/properties	Enunciation; analysis– synthesis; generalization	$Dom(f)$ $\leftrightarrow Ran(f^{-1}); Ran(f)$ $\leftrightarrow Dom(f^{-1})$
C22	Structural comparison	Reversibility	Graphs, concepts, propositions/properties	Analysis–synthesis; argumentation; generalization; representation– meaning	Structural properties of exponential function $\leftrightarrow$ corresponding properties of logarithmic function
C23	Logical or implicative reasoning	Implication	Propositions, arguments, language	Enunciation; argumentation; generalization– particularization	Premise/condition $\rightarrow$ mathematical consequence

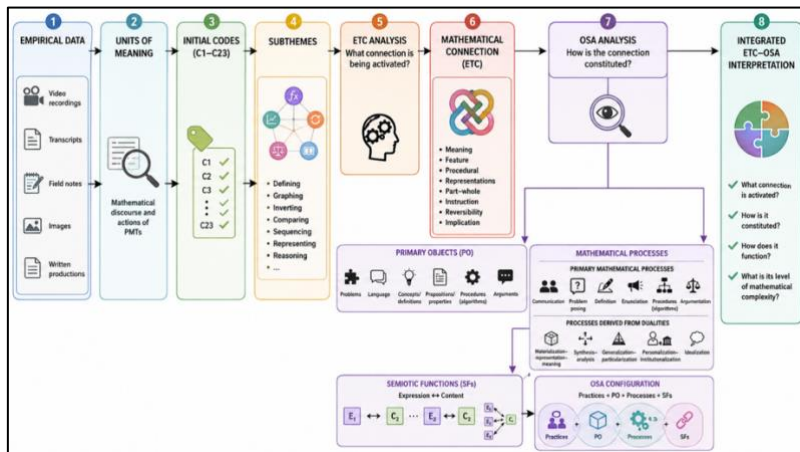
Source: Authors' own elaboration.

Table 1 makes the ETC+OSA networking operational by connecting the empirical and theoretical levels of analysis. Codes represent empirical evidence extracted from the PMTs' mathematical activity; subthemes synthesize recurrent patterns of action; and ETC themes classify these patterns according to the type of mathematical connection activated. The OSA level subsequently decomposes each connection into the primary objects and mathematical processes involved and the semiotic functions relating these objects. Thus, the columns should not be interpreted as independent classifications, but as successive analytical layers through which an empirical mathematical action is progressively reconstructed as a mathematical connection and, subsequently, as an onto-semiotic configuration.

In **Table 1**, empirical data were first segmented into units of meaning, from which codes (C1–C23) and subthemes were generated. These patterns were then analyzed through ETC to identify and classify the mathematical connections activated. Subsequently, OSA provided a deeper analytical level by reconstructing the underlying mathematical practices and identifying the primary objects (PO), mathematical processes, and semiotic functions (SFs) involved. Semi-symbolic expression–content relationships were used to represent how mathematical objects become interconnected and acquire meaning. Thus, ETC and OSA were applied as successive and complementary analytical layers, rather than as parallel frameworks: ETC identifies what type of connection is activated, while OSA explains how that connection is constituted and functions. The resulting pathway is: data $\rightarrow$ codes $\rightarrow$ ETC connection $\rightarrow$ OSA configuration $\rightarrow$ integrated interpretation (see **Figure 2**).

Figure 2

Complete analytical pathway: empirical data → units of meaning → codes → subthemes → ETC connection type → OSA configuration of practices, primary objects, processes, and semiotic functions



Phase 5: Definition, refinement, and onto-semiotic interpretation of themes

In this phase, the themes identified in the previous stages were conceptually delimited and refined to establish clear, coherent, and theoretically grounded definitions according to the ETC. Each theme was therefore interpreted as a type of mathematical connection emerging from recurrent relationships among mathematical objects, procedures, properties, meanings, and representations evidenced in the PMTs' practices. Subsequently, the OSA was used as a complementary analytical layer to unpack the internal structure and functioning of these connections. This involved identifying the mathematical practices, primary objects (PO), mathematical processes, and semiotic functions (SFs) underlying each connection. Accordingly, the themes were not treated merely as descriptive categories, but as relational structures whose onto-semiotic configuration made it possible to explain how the identified connections were constituted and the level of complexity achieved in the PMTs' mathematical activity.

The meaning connection comprised practices through which PMTs attributed meaning to exponential and logarithmic functions by articulating formal definitions, symbolic expressions, structural elements, and conceptual explanations, particularly the interpretation of logarithmation as the process of determining an exponent. From the OSA perspective, this connection involved primary objects such as language, concepts/definitions, propositions, and procedures, together with processes of communication, definition, representation–meaning, analysis, and particularization. The associated semiotic functions linked symbolic and verbal expressions with their mathematical content, making explicit what the exponential and logarithmic objects represent. Nevertheless, these meanings remained predominantly within the formal mathematical domain.

The feature connection was characterized by identifying and relating properties that distinguish exponential and logarithmic functions, including domain, range, base, exponent, and graphical behavior. Onto-semiotically, this connection involved concepts, propositions/properties, and graphical and symbolic language, activated mainly through processes of communication, enunciation, analysis, generalization, and particularization. Semiotic functions linked representations of the functions with their corresponding properties. Although PMTs correctly identified these features, they were sometimes presented separately rather than integrated into arguments supporting a systemic understanding of the functions.

The procedural connection comprised the relationships established through rules, algorithms, and operational sequences, including substitution, application of algebraic properties, calculation of function values, and construction of tables. From the OSA perspective, procedures constitute primary objects while procedures (algorithms), particularization, materialization–representation, and analysis–synthesis emerge as relevant mathematical processes. For example, substituting particular values into a general expression generates numerical outputs that can subsequently be organized into a table and represented graphically. Although these practices demonstrated operational competence, their articulation with conceptual meanings and argumentation was sometimes limited.

The different representations connection involved coordinating symbolic, numerical, tabular, graphical, and verbal representations associated with the same mathematical object. Rather than merely identifying the presence of several representations, the analysis focused on the relationships established among them. From the OSA perspective, this connection activated language in different semiotic forms and processes of communication, materialization–representation–meaning, analysis–synthesis, and particularization–generalization. Semiotic functions made explicit the correspondences between algebraic expressions, numerical values, tables, and graphs.

Thus, the strength of this connection depended not on the number of representations used, but on the extent to which PMTs established meaningful relationships among them.

The part-whole connection was interpreted as the relationship between particular components of a mathematical object and the global structure to which they belong. It emerged especially during graph construction, where ordered pairs, key points, asymptotes, and segments of the curve contributed to constructing and interpreting the graph as a whole. Onto-semiotically, this connection involved graphical language, concepts, propositions/properties, and procedures, together with processes of analysis-synthesis, particularization-generalization, procedures, and materialization-representation. Analysis allowed particular components to be distinguished, whereas synthesis supported their integration into the global representation. However, the relationship often remained operational rather than developing into a fully structural interpretation of the function.

The instruction-oriented connection encompassed practices through which PMTs related previously learned mathematical knowledge to new content and organized mathematical ideas through prerequisites, sequencing, and recapitulation. From the OSA perspective, this connection involved concepts, propositions, procedures, and arguments mobilized through communication, personalization-institutionalization, analysis-synthesis, and representation-meaning processes. Prior personal or previously institutionalized meanings were reactivated and connected with new institutional meanings during instruction. Although these practices revealed a clear didactic intention, their potential to organize and promote other mathematical connections was not always fully exploited.

The reversibility connection was characterized as a bidirectional relationship that enables movement from a mathematical object, process, or representation to another and back again. In this study, it was particularly evident in the inverse relationship between exponential and logarithmic functions. Its manifestation ranged from recognizing that both functions are inverses to performing algebraic inversion, interchanging ordered pairs, relating domain and range, and comparing structural and graphical properties. From the OSA perspective, this connection mobilized concepts, propositions, procedures, representations, and arguments through processes of enunciation, procedures (algorithms), analysis-synthesis, generalization-particularization, materialization-representation-meaning, and argumentation. Semiotic functions established correspondences such as $a^x = b$ and $\log_a b = x$. Consequently, reversibility exhibited increasing levels of complexity, progressing from recognition and procedural transformation toward a more structural coordination of meanings, properties, and representations. This progression is already evidenced in the coding structure from C17 to C22.

Finally, the implication connection was defined through logical relationships in which mathematical conditions or premises lead to corresponding consequences, typically expressed through explicit or implicit “if-then” structures. From the OSA perspective, this connection primarily involved propositions and arguments and activated processes of enunciation, argumentation, analysis, generalization, and particularization. The associated semiotic functions linked antecedent conditions with their mathematical consequences. Therefore, implication played an important role in providing argumentative support for other connections, particularly when PMTs attempted to justify properties, procedures, or the reversibility between exponential and logarithmic functions.

Overall, this phase established an explicit analytical bridge between ETC and OSA. ETC provided the categories for identifying and classifying the mathematical connections activated by the PMTs, whereas OSA made it possible to reconstruct their internal organization in terms of mathematical practices, primary objects, processes, and semiotic functions. Consequently, the integrated analysis moved beyond determining whether a connection was present to explaining how it was constituted, how it functioned, which mathematical processes sustained it, and what level of structural complexity it achieved. This distinction strengthens the methodological complementarity between the two theoretical frameworks and prevents their use as parallel or overlapping analytical systems.

Phase 6: Report production

In this phase, the results were systematically organized and interpreted by articulating the themes identified through the ETC with the complementary analytical tools of the OSA. The analysis showed not only which mathematical connections were activated by the PMTs, but also how these connections were constituted and functioned through mathematical practices, primary objects, processes, and semiotic functions. Illustrative excerpts from classroom discourse, written productions, graphical representations, and semi-symbolic expression-content relationships were used as empirical evidence to support the interpretations. Particular attention was given to the coordination among meanings, procedures, properties, and representations, as well as to the increasing complexity of the reversibility connection between exponential and logarithmic functions. Finally, the findings were discussed in relation to previous research (Campo-Meneses & García-García, 2020, 2021), allowing similarities and differences to be identified regarding the activation, coordination, and difficulties associated with mathematical connections, particularly representation, procedural, and reversibility connections.

RESULTS

The findings are presented according to the thematic organization derived from the analysis in Phase 3. Across the two cases, the meaning connection was activated through formal definitions and conceptual explanations, although its configuration differed in emphasis. PMT1 primarily articulated the exponential function through its symbolic structure and graphical behavior, whereas PMT2 emphasized the logarithmic function and its inverse relationship with exponentiation. Cross-case analysis therefore showed a shared reliance on formal mathematical meanings, but different pathways for constructing and communicating those meanings. The formal results correspond to the eight themes included in the C1–C23 coding matrix; the observation presented in the subsection on metaphorical connections is reported separately as emergent and exploratory and does not constitute an additional coded theme.

Instruction-oriented connection

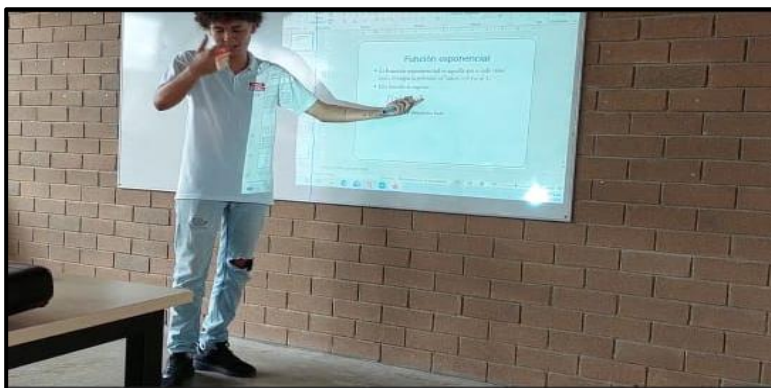
The instruction-oriented connection functioned as an organizing structure of the PMTs' didactic-mathematical activity by linking prior knowledge with new mathematical content through prerequisites, sequencing, and recapitulation. The PMTs drew on previously institutionalized meanings related to powers, exponents, and functions and progressively connected them with definitions, examples, tables, and graphs of exponential and logarithmic functions. From an onto-semiotic perspective, this connection mobilized concepts, propositions, procedures, and arguments through processes of communication, personalization–institutionalization, analysis–synthesis, and representation–meaning. The corresponding semiotic functions linked prior mathematical knowledge with the meanings of the new objects, while sequencing connected ordered mathematical actions with the overall instructional organization. Thus, the connection reveals not only a didactic intention to organize content, but also a process of reconstructing institutional meanings.

Meaning connection

The meaning connection emerged as a central component of the PMTs' mathematical activity, as they attributed meaning to exponential and logarithmic functions by articulating formal definitions, symbolic expressions, structural elements, and conceptual explanations. From an onto-semiotic perspective, this connection mobilized language, concepts/definitions, propositions, and procedures through processes of communication, definition, representation–meaning, analysis, and particularization. As illustrated in [Figure 3](#), PMT1 introduced the exponential function through its general symbolic expression, identifying its base, exponent, and corresponding restrictions. PMT1 introduced the exponential function as a relationship of the form $f(x) = a^x$, where the base $a > 0$ and $a \neq 1$, emphasizing its functional structure and core components.

Figure 3

Explanation of PMT1 and connection of meaning

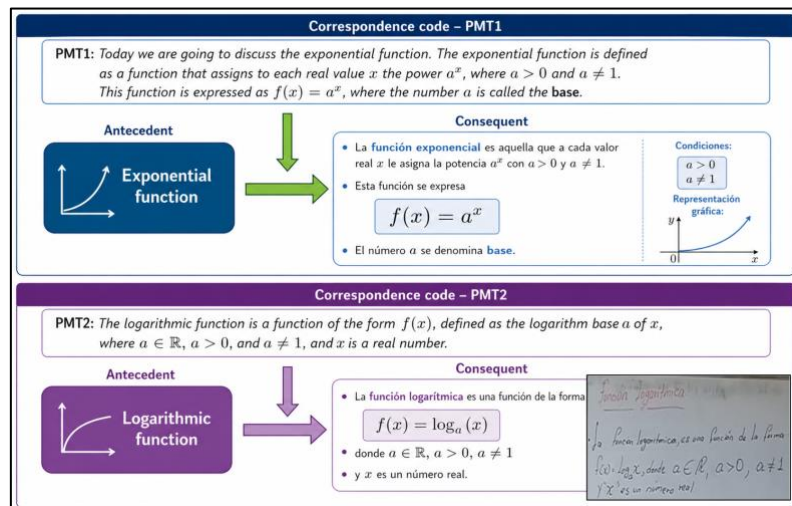


Source: Authors' own elaboration.

The construction of meaning was further developed through the articulation of both exponential and logarithmic functions. In particular, the logarithmic function was presented as $f(x) = \log_a(x)$, highlighting its role as the inverse process of exponentiation. As shown in [Figure 4](#), both PMTs established relationships between these functions, linking definitions with their corresponding meanings through explanatory discourse, in line with the scheme proposed by Rodríguez-Nieto et al. (2024).

Figure 4

Mathematical connections of meaning of PMT1 and PMT2 (semiotic functions).

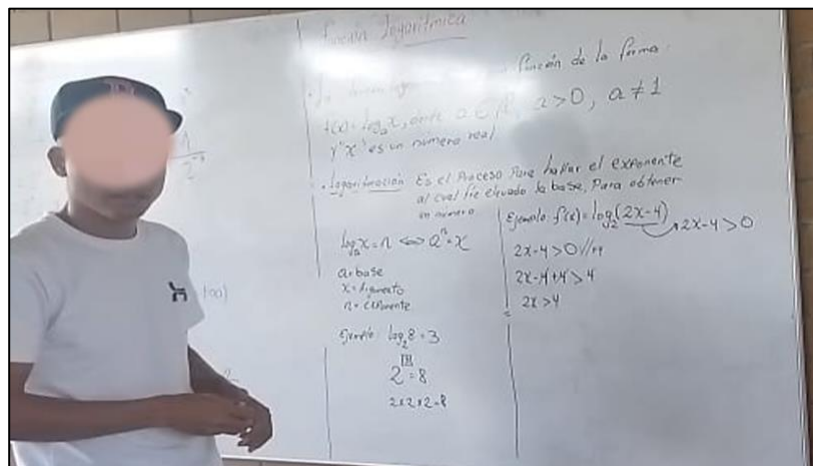


Source: Authors' own elaboration.

This understanding was reinforced through explicit explanations of logarithms as the process of determining the exponent to which a base must be raised to obtain a given value. These explanations integrated key elements such as base, exponent, and argument, contributing to a more structured interpretation of the concepts. As evidenced in **Figure 5**, these meanings were also developed through board-based explanations and examples, which supported the articulation between symbolic expressions and verbal reasoning.

Figure 5

Connection of meaning written on the board and its application

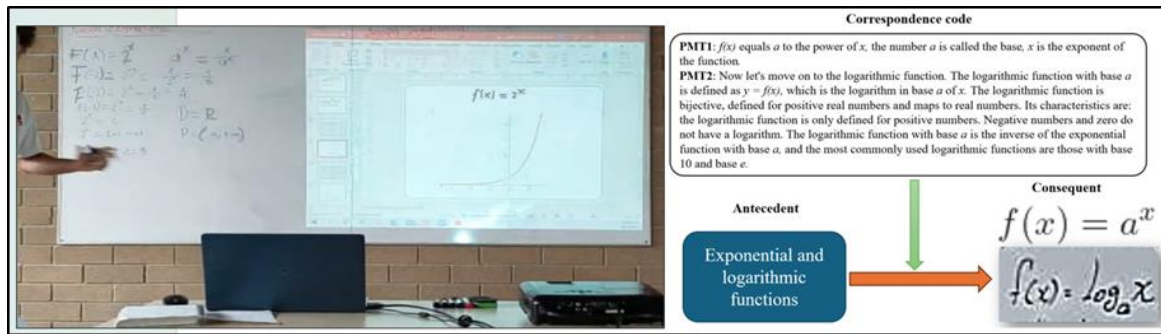


Source: Authors' own elaboration.

Although the PMTs went beyond merely stating formal definitions and attempted to make the concepts more accessible, the analysis shows that these meanings remained largely within the formal mathematical domain, with limited connection to real-world contexts. From an onto-semiotic perspective, this reflects a predominance of institutional meanings, with limited construction of personal meanings, which restricts the transferability and functionality of knowledge in other contexts.

Feature Connection

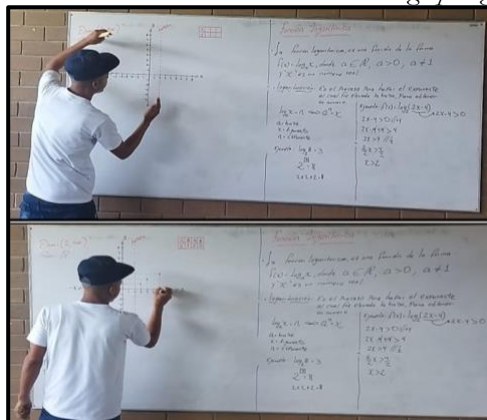
The feature connection emerged when the PMTs identified and related properties that characterize the structure and behavior of exponential and logarithmic functions. From an onto-semiotic perspective, this connection mobilized concepts, propositions/properties, and symbolic, numerical, and graphical language through processes of communication, enunciation, analysis, generalization, and particularization. As illustrated in **Figure 6**, PMT1 analyzed the exponential function by constructing a table of values and its corresponding graph, relating changes in the exponent to variations in the function and emphasizing properties such as exponential growth and the role of the base. These relationships established semiotic functions between numerical and graphical representations and the properties they signify, allowing particular values and graphical behaviors to be interpreted as manifestations of more general characteristics of the exponential function.

Figure 6*Evidence of the feature connection and semiotic function**Source: Authors' own elaboration.*

In addition, PMT1 emphasized fundamental elements of the exponential function, such as the base and exponent, contributing to the understanding of its structure. Similarly, PMT2 described the logarithmic function by identifying properties such as its domain restricted to positive real numbers, its inverse relationship with the exponential function, and its dependence on the base. PMT2 also referred to commonly used bases, such as 10 and e , enabling comparisons between different types of logarithmic functions. However, although these properties were correctly identified, they were primarily presented in a descriptive manner and were not consistently integrated into deeper conceptual explanations.

Procedural connection

The procedural connection emerged through the coordinated use of algorithms, substitutions, algebraic rules, and sequential calculations to obtain and represent values of exponential and logarithmic functions. From an onto-semiotic perspective, this connection mobilized procedures, symbolic and numerical language, concepts, and propositions, mainly through processes of procedures (algorithms), particularization, analysis–synthesis, and materialization–representation. As illustrated in Figure 7, PMT2 substituted specific values into the logarithmic expression, evaluated the corresponding logarithms, and organized the resulting numerical outputs into a table. These procedures subsequently supported the construction of ordered pairs and the graphical representation on the Cartesian plane. Thus, the activated semiotic functions linked algebraic expressions, numerical results, tabular values, and graphical points, showing how procedural activity served as a bridge between calculation and representation, although its conceptual integration depended on the explicit justification of the relationships among these objects.

Figure 7*Procedural mathematical connections and graphing functions**Source: Authors' own elaboration.*

Similarly, PMT1 applied procedures when working with the exponential function, using substitution and exponent rules to compute values and describe the behavior of the function. This included handling negative exponents, evaluating expressions, and identifying general patterns such as growth and range.

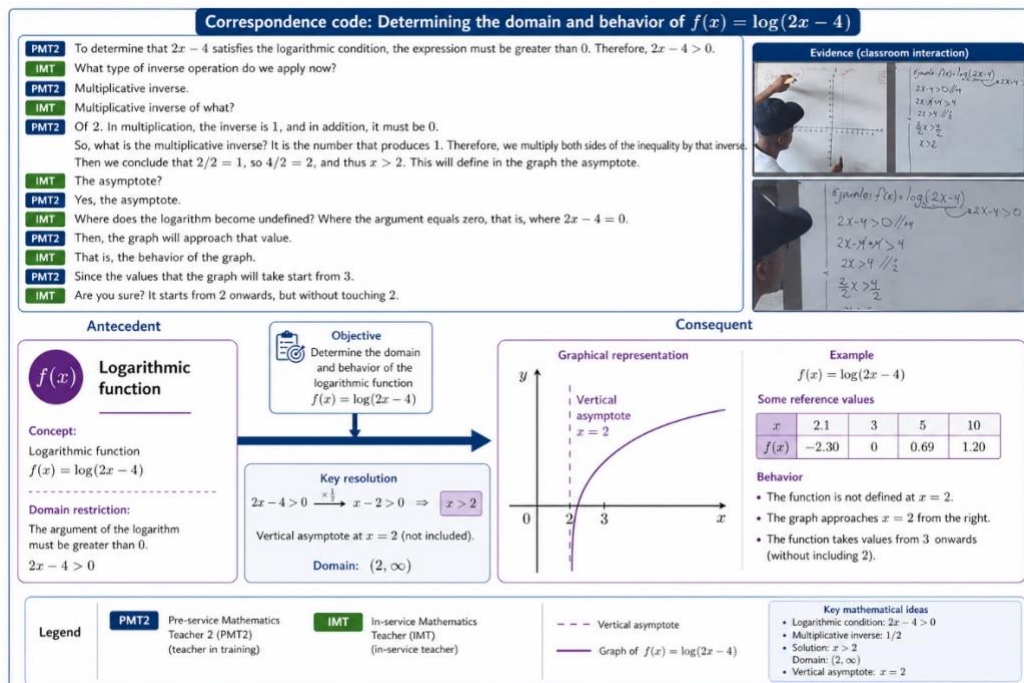
Overall, procedural connection was one of the most prominent in the observed practice, reflecting a strong operational command of mathematical techniques. However, these procedures were predominantly instrumental, as they were often executed without explicit conceptual justification. This reveals a limited articulation between procedural and conceptual knowledge, where emphasis is placed on correct execution rather than on understanding underlying processes, consistent with previous research (García-García & Dolores-Flores, 2021).

Part-whole connection

The part-whole connection emerged when the PMT's related specific components of a function to its overall graphical structure, particularly key points and asymptotes involved in the construction and interpretation of the logarithmic function. From an onto-semiotic perspective, this connection mobilized concepts, propositions, graphical language, and procedures through processes of analysis-synthesis, particularization-generalization, procedures, and materialization-representation. As illustrated in Figure 8, PMT2 identified the asymptote from the corresponding restriction and related particular points of the function to the construction of its graph. The associated semiotic functions linked these local elements to the overall graphical configuration, showing how particular components contribute to understanding the function as a whole (García-García & Dolores-Flores, 2021).

Figure 8

Part-whole mathematical connections.

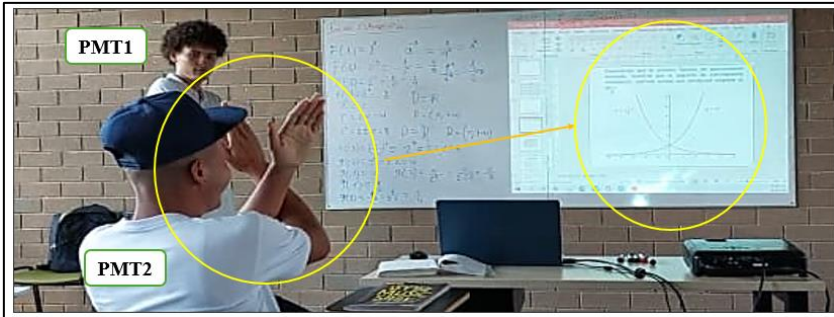


Source: Authors' own elaboration.

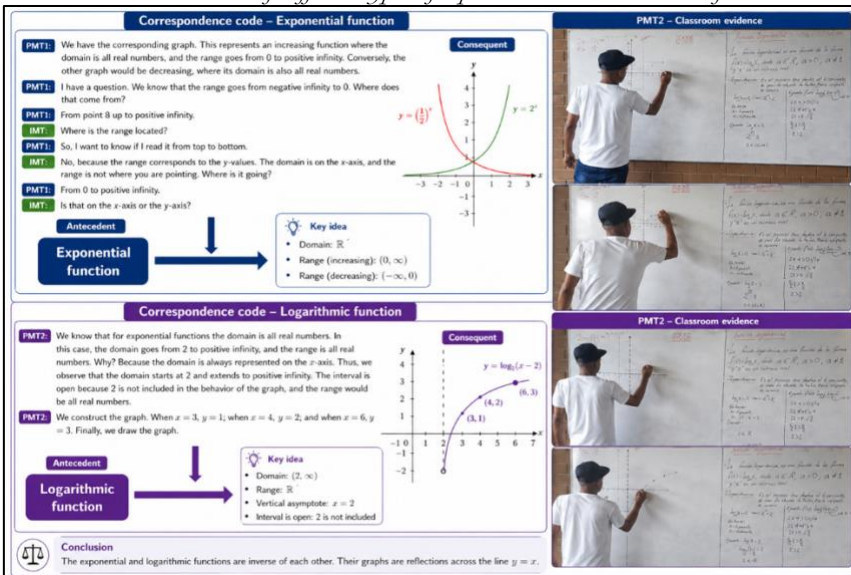
More broadly, this connection was evident during graph construction, where PMT's organized the process into stages, such as plotting the Cartesian plane, determining points through tables of values, and connecting these points to form the curve. This indicates a structured approach in which partial elements contribute to building a global representation. However, the analysis shows that this connection remained primarily at an operational level. Although the parts were identified and used coherently, their relationship with global properties such as asymptotic behavior, continuity, or overall trend was not explicitly explained. Consequently, the part-whole relationship was not consolidated as a structural understanding, but rather as a sequence of procedures.

Different representations connections

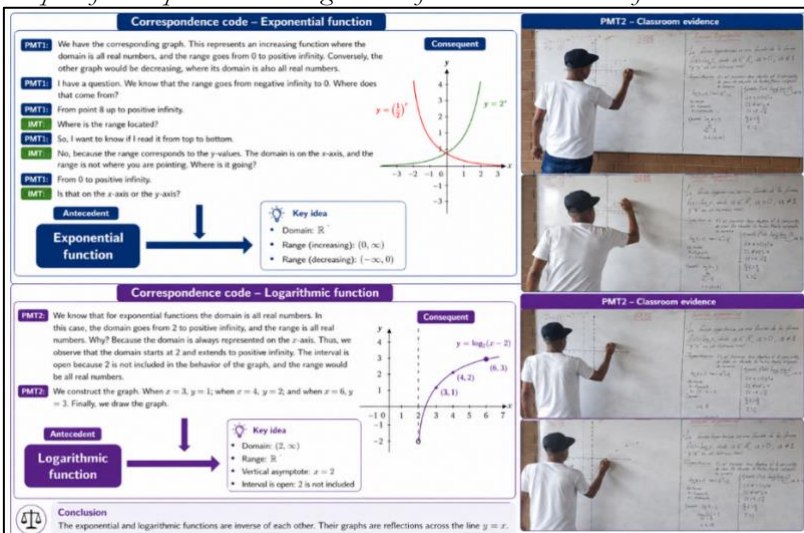
The different representations connection emerged when the PMT's coordinated verbal, symbolic, numerical, tabular, and graphical representations of the same mathematical object, rather than merely using multiple representations independently (Businskis, 2008). From an onto-semiotic perspective, this connection mobilized different forms of mathematical language, concepts, and procedures through processes of communication, materialization-representation-meaning, analysis-synthesis, and generalization-particularization. As illustrated in Figure 9, PMT1 related symbolic expressions to their graphical representations to interpret the behavior of the functions and identify properties such as growth and symmetry. The associated semiotic functions established correspondences among expressions, representations, and mathematical meanings, showing that the quality of this connection depends not on the number of representations used, but on the extent to which the PMT's meaningfully coordinate and translate among them.

Figure 9*Mathematical connections of the different representations of PMT1**Source: Authors' own elaboration.*

This use of multiple registers was further developed through the construction of tables of values and their translation into graphical representations. As shown in **Figure 10**, PMTs were able to move between representations, linking algebraic expressions, numerical values, and graphs.

Figure 10*Mathematical connections of different types of representations and semiotic functions**Source: Authors' own elaboration.*

Finally, the integration of procedures and representations resulted in complete graphical constructions, where the relationships between different registers became visible (**Figure 11**).

Figure 11*Graphs of the exponential and logarithmic functions and semiotic functions.**Source: Authors' own elaboration.*

Despite this use of multiple representations, the analysis shows that their coordination was limited. Although PMT's translated information between registers, they did not consistently establish deep explanatory relationships that integrated these representations into a coherent system of meanings. From an onto-semiotic perspective, this indicates a weak articulation of semiotic functions, which restricts the development of a comprehensive understanding of the mathematical object.

Exploratory observation related to metaphorical connections

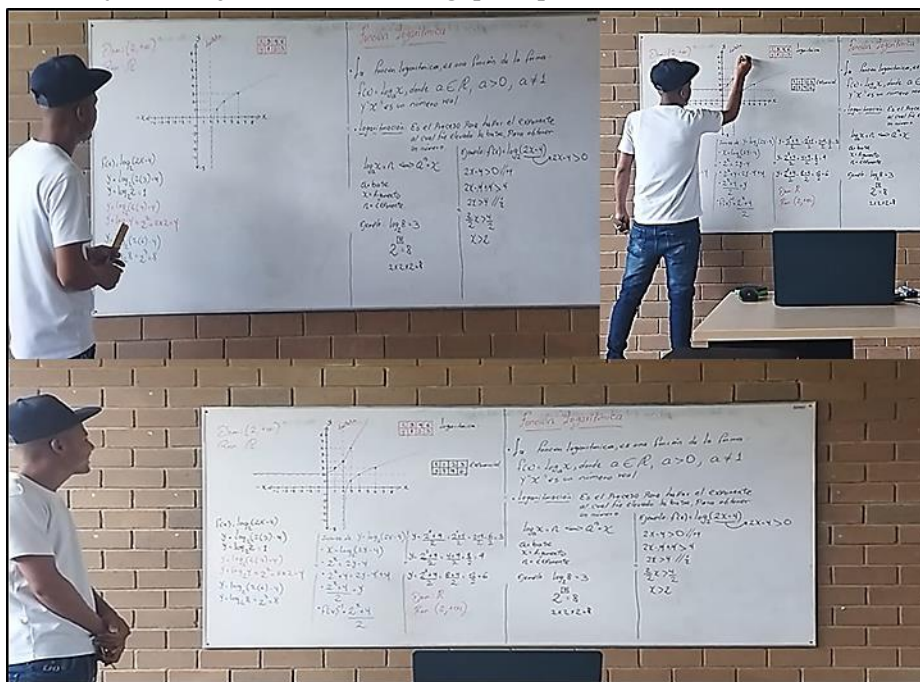
Outside the formal C1–C23 coding matrix, an exploratory observation was made when the PMT used bodily gestures to give perceptible meaning to the behavior and shape of an abstract mathematical object. In particular, the movement and positioning of the hands represented the shape and behavior of the projected graph (a parabola or exponential function), establishing a correspondence between a concrete bodily domain and the abstract domain of the function. From the ETC perspective, expressions such as “the graph opens”, “rises”, or “expands” may be interpreted as a conceptual metaphor in which the graph is understood as a physical object or path. From an onto-semiotic perspective, the gesture may be interpreted as ostensive language that materializes a non-ostensive mathematical object through processes of communication, materialization–representation–meaning, and idealization. The associated semiotic function may connect the bodily gesture (expression) with the graphical behavior of the parabola or exponential function (content), showing how embodied and visual resources can support the construction and communication of mathematical meaning. This observation was not included as a formal theme in the coding matrix and should therefore be understood as emergent and exploratory.

Reversibility connection (key finding)

The reversibility connection emerged as the most structurally complex connection, particularly in PMT2's explanation of the inverse relationship between exponential and logarithmic functions (García-García & Dolores-Flores, 2021). From an onto-semiotic perspective, it mobilized concepts, propositions, procedures, representations, and arguments through processes of analysis–synthesis, representation–meaning, procedures, and argumentation. As illustrated in Figure 12, reversibility was manifested through the recognition of both functions as inverses and through bidirectional relationships between their algebraic forms, ordered pairs, domain–range relationships, and graphical properties. These manifestations were organized into recognition, procedural, and structural forms, reflecting different degrees of coordination among mathematical meanings and representations.

Figure 12

Evidence of reversibility connections and their graphical potential



Source: Authors' own elaboration.

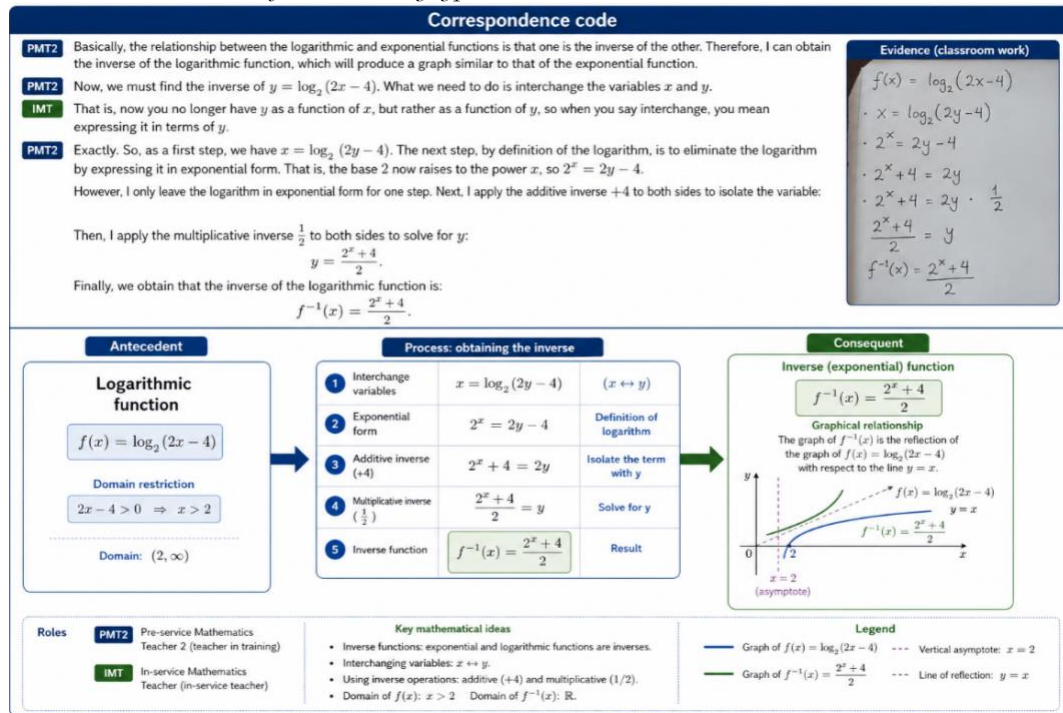
PMT2 demonstrated reversibility by transforming a logarithmic expression into its exponential form, interchanging variables, and solving algebraically to obtain the inverse function. This process was further validated through numerical verification and the analysis of ordered pairs, showing that if a point (x, y) belongs to a function, then (y, x) belongs to its inverse. In addition, PMT2 identified the inversion of domain and range, as

well as structural changes such as the transformation of a vertical asymptote into a horizontal one. These elements indicate that reversibility was not limited to procedural transformations but involved the coordination of algebraic, numerical, and graphical representations.

As shown in **Figure 13**, this articulation allowed for the visualization of inverse relationships between functions, suggesting an emerging understanding of their structural correspondence.

Figure 13

Mathematical connections of the reversibility type



Source: Authors' own elaboration.

From an analytical perspective, reversibility goes beyond reversing steps; it involves recognizing that mathematical transformations preserve a structure that allows the original expression to be recovered through inverse operations. In this sense, the connection reflects a bidirectional and coherent system of relationships. Moreover, the findings suggest that reversibility may function as an integrative connection, coordinating procedural, representational, feature, and implication connections. However, this integration is not fully consolidated, as the connection remains predominantly procedural in many cases, with only partial development toward a structural understanding.

Implication connection

The implication connection emerged through logical “if-then” relationships linking mathematical conditions with their consequences, particularly when determining the domain of logarithmic functions from inequalities or explaining functional behavior according to the base. From an onto-semiotic view, this connection mobilized propositions and arguments through processes of enunciation, argumentation, analysis, and generalization, establishing semiotic functions between mathematical conditions and their consequences. Implication was also closely articulated with reversibility: while reversibility establishes a bidirectional relationship between exponential and logarithmic forms, implication provides the logical justification for that transformation. Thus, when PMT2 states that “if $\log_a x = y$, then $a^y = x$ ”, both connections are activated: reversibility enables the inverse transformation, whereas implication validates its coherence under the corresponding mathematical conditions.

Onto-semiotic interpretation of connections

Table 2 in the APPENDIX, presents the operationalization of the mathematical connections identified as onto-semiotic configurations, in which each code is interpreted as a mathematical practice that articulates semiotic processes, objects, and functions, allowing us to show that the connections are not isolated entities, but systems of relationships that structure didactic mathematical knowledge.

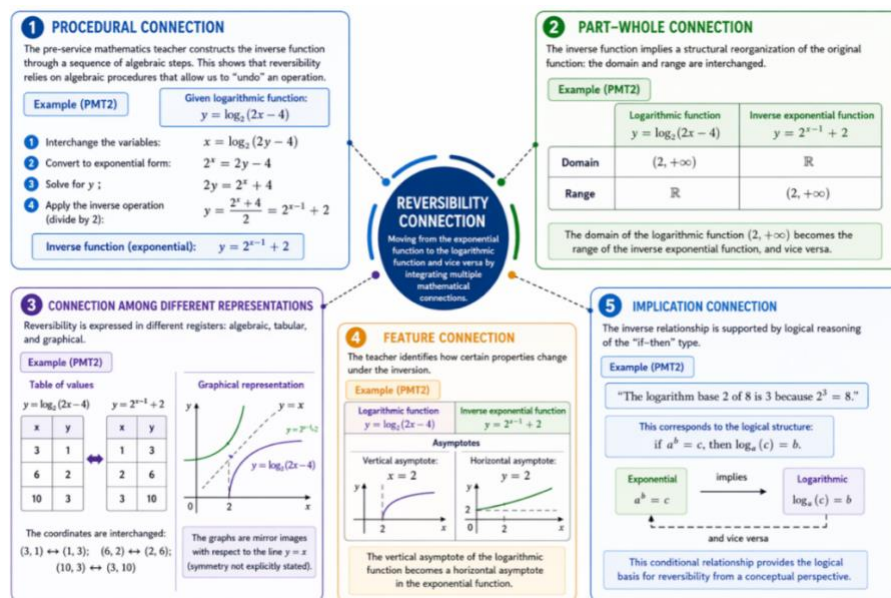
Based on the results, the reversibility connection can be understood as a complex relational structure that does not emerge in isolation but through the articulation of multiple mathematical connections. In this sense, transitioning between exponential and logarithmic functions requires the integration of different dimensions of

mathematical knowledge, as evidenced in the PMTs' practices. Specifically, reversibility is supported by: (i) procedural connections, through algebraic transformations that allow operations to be reversed; (ii) part-whole connections, when recognizing the reorganization of elements such as domain and range; (iii) representational connections, expressed in the correspondence of ordered pairs and graphical relationships; (iv) feature connections, through the identification of structural changes, such as asymptotes; and (v) implication connections, which provide the logical structure supporting inverse relationships.

As illustrated in **Figure 14**, the articulation of these connections suggests that reversibility may function as an integrative connection that coordinates procedures, properties, representations, and reasoning.

Figure 14

Details of the reversibility connection based on other mathematical connections



Source: Authors' own elaboration.

However, the results show that this articulation is not fully consolidated, as reversibility is predominantly approached from a procedural perspective, without explicit integration of the other connections. This limits its potential to support a deeper understanding of the relationship between exponential and logarithmic functions.

Reversibility as a potentially integrative connection

The findings of this study suggest that reversibility may function as an integrative connection. This is because, in the analyzed episodes, it does not act in isolation but coordinates other connections: procedural, representational, feature, and implication connections. For example, the role of the implication connection in "if...then" structures details how this connection provides the logical and argumentative support that validates and legitimizes the reversible transformation $((\log_a(x) = y), \text{ then } (a^y = x))$. This approach positions reversibility as one of the most complex relational structures observed in the two cases, since it requires coordinating logical reasoning with algebraic procedures and graphical or numerical representations.

Analytical forms of reversibility observed in the two cases

The analysis identified three forms through which reversibility was manifested in the mathematical activity of the two participants:

Recognition Form: The participant states verbally or theoretically that the logarithmic function is the inverse of the exponential function. This form corresponds to the identification code C17, without further explanations, justifications, or applications of the mathematical implications of this inverse relationship.

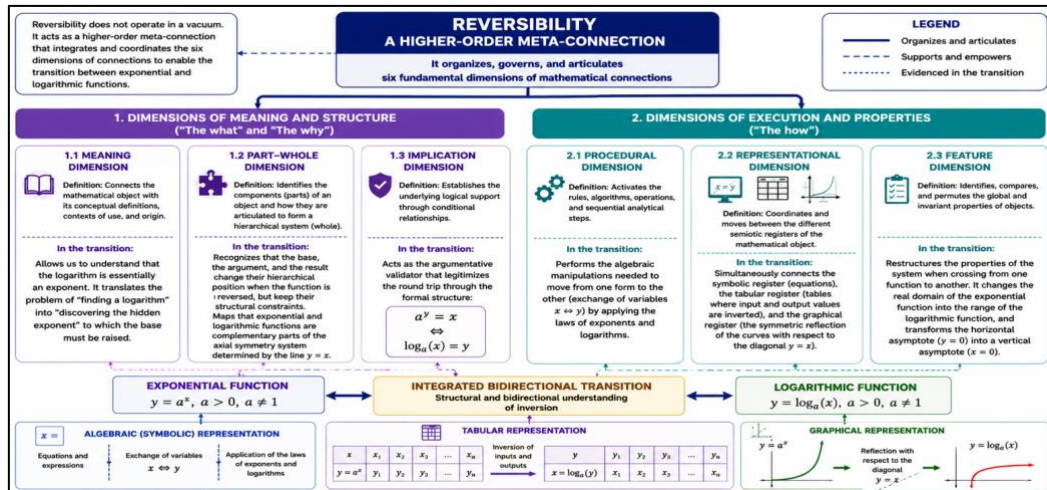
Procedural Form: The participant activates technical and algorithmic skills and demonstrates reversibility by executing structured algebraic steps: taking the analytical expression, performing the variable interchange $((x \leftrightarrow y))$, and applying algebraic manipulation rules to derive the inverse function.

Structural Form: This was the most complex form observed in the analyzed episodes. The participant not only performs calculations but also coordinates and compares properties of both mathematical objects: recognizes that ordered pairs are interchanged from $((x, y))$ to $((y, x))$, relates the domain of one function to the range of its inverse, and connects the vertical asymptote of the logarithmic function with the horizontal asymptote of the exponential function.

To further develop the analysis based on the ETC, reversibility is interpreted as a potentially integrative connection structured across six dimensions (see Figure 15).

Figure 15

Mathematical dimensions supporting reversibility in the analyzed episodes.



Source: Authors' own elaboration.

DISCUSSION

The findings of this study address the research objective by revealing not only the diversity of intra-mathematical connections activated by two PMTs when explaining exponential and logarithmic functions, but also the different levels of depth, coordination, and structural integration through which these connections emerge in instructional discourse. Beyond identifying instruction-oriented, meaning, feature, part-whole, procedural, implication, reversibility, and representational connections, the main contribution lies in showing that these relationships do not operate as isolated categories; rather, they form an interconnected system of mathematical practices in which concepts, procedures, properties, representations, and arguments are coordinated through semiotic functions.

The combined ETC+OSA perspective therefore extends the analysis from the mere presence of connections to their quality, function, and interdependence, providing a more relational account of the didactic-mathematical knowledge mobilized by pre-service teachers. Particularly significant is the role of reversibility, which emerged as the most structurally complex connection because the transition between exponential and logarithmic functions required the coordination of meaning, procedural, representational, feature, part-whole, and implication connections. Thus, the findings suggest that reversibility may function as a potentially integrative connection that organizes and articulates other intra-mathematical relationships, enabling a bidirectional and progressively more structural understanding of exponential and logarithmic functions.

Analyzing the results reveals significant differences in how participants mobilize mathematical connections. For example, while PMT1 shows a greater emphasis on procedural and representational connections, focused on constructing tables and graphs, PMT2 demonstrates a more advanced development of the reversibility connection, articulating algebraic processes, numerical correspondences, and structural relationships between inverse functions. However, in both cases, the connections of meaning and feature tend to remain at a descriptive level, without fully integrating into explanations that allow for inferring global behaviors. Similarly, the implication connection appears implicitly, which limits the consolidation of more robust mathematical arguments. This comparison demonstrates that the quality of connections depends not only on their activation but also on their level of articulation and coherence within the system of mathematical relationships.

In contrast to previous research, the results of this study not only confirm trends reported in the literature but also allow for a deeper exploration of them from a situated didactic perspective. For example, studies focused on modeling (Araújo et al., 2022; Loi et al., 2020) highlight its potential to foster the understanding of exponential and logarithmic functions; however, the findings of this research demonstrate that, even in teaching contexts without an emphasis on modeling, pre-service teachers are able to activate mathematical connections, although predominantly at descriptive and procedural levels. Likewise, consistent with Campo-Meneses and García-García (2020, 2023, 2025) and Campo-Meneses et al. (2021), difficulties are identified in the articulation between representations and in the understanding of the relationship between exponential and logarithmic functions; however, this study expands upon these results by showing that these limitations persist even among pre-service teachers in teaching situations. Similarly, although research such as Campo-Meneses et al. (2024) analyzes criteria

that guide teaching practice, the results obtained here show that these criteria do not always translate into an effective articulation of mathematical connections in discourse.

Furthermore, in line with studies on technology use (Koştur & Yılmaz, 2017; Porras, 2025; Silva, 2026), it is recognized that digital resources can enhance understanding; however, this study suggests that their impact is limited if not accompanied by a conceptual and argumentative articulation of the connections. Likewise, the results coincide with those reported by Okoye-Ogbalu and Nnadozie (2024), who demonstrate difficulties among future teachers in using logarithmic rules, but broaden this perspective by showing that these difficulties are not only operational but also structural, affecting key connections such as reversibility. In this sense, the main novelty of this study lies in analyzing mathematical connections in teaching practice, demonstrating that it is not enough to identify their presence, but rather it is necessary to understand their level of articulation, their function within the discourse, and their integration into a system of relationships. Furthermore, the articulation between ETC and OSA allows us to propose that mathematical connections should be understood as interdependent semiotic configurations, which constitutes a distinctive contribution compared to previous studies focused mainly on their identification or classification.

From a theoretical perspective, it is evident that mathematical connections can be interpreted as semiotic functions (OSA) that organize the relationships between mathematical objects in discourse. In this sense, it is proposed that the quality of didactic-mathematical knowledge can be analyzed based on the degree of integration between connections, moving from descriptive to structural and argumentative levels. This approach opens the possibility of developing a more robust analytical framework for evaluating mathematics teaching, one that considers not only which connections are activated, but also how they are articulated, justified, and used to construct meaning.

It should be noted that Campo-Meneses et al. (2021) did not consider metaphorical connections because their analysis focused solely on students' mathematical activity and not on the teaching practices through which mathematical meanings are communicated. In contrast, an exploratory observation in the present study suggests that metaphorical resources may emerge in the discourse and bodily actions of a PMT when explaining exponential and logarithmic functions. In particular, gestures associated with growth, direction, shape, or graphical behavior may function as semiotic resources that project bodily and visual experiences onto abstract mathematical objects. From the ETC+OSA perspective, this observation suggests that metaphorical resources may act as mediators of meaning during teaching by coordinating verbal, gestural, graphical, and symbolic resources. Although this observation was not part of the formal coding matrix, it opens a possible line of research on how metaphorical and bodily resources may support the construction and communication of mathematical meanings in initial teacher education.

This study should be interpreted within the boundaries of an exploratory comparative case-study design. The two participants were purposively selected as information-rich, high-performing cases and therefore should not be considered representative of the broader population of pre-service mathematics teachers. Consequently, the findings are not intended for statistical generalization. Their contribution lies instead in their analytical transferability: the detailed ETC–OSA characterization of the mathematical connections identified in these cases provides theoretical and methodological insights that may be examined in comparable teacher-education contexts.

CONCLUSIONS

This study concludes that, in the two cases analyzed, mathematical connections were not activated as isolated relationships but as an interconnected system within the PMTs' explanations of exponential and logarithmic functions. Reversibility showed the highest degree of structural integration, as it coordinated algebraic procedures, representations, properties, and logical relationships involved in understanding the inverse relationship between both functions. These findings suggest that examining not only the presence of mathematical connections but also their degree of articulation provides situated evidence for analyzing the didactic-mathematical knowledge mobilized by pre-service mathematics teachers. However, this work has some limitations. The study focused solely on two PMTs and the analysis of one class, which requires caution when considering the transferability of the findings to other participants and instructional contexts. Furthermore, while the established connections were identified, their direct impact on student learning was not evaluated. Although the prompts were open-ended and limited to ideas already introduced by the participants, their use may have influenced the degree of elaboration and explicitness of some mathematical connections. Therefore, not all the observed elaborations can necessarily be considered entirely spontaneous. Future research could expand the sample of participants, include different educational levels, and design interventions that specifically strengthen complex connections such as reversibility. It would also be pertinent to explore how integrating theoretical frameworks such as ETC and OSA (Rodríguez-Nieto & Font, 2025) may contribute to teacher education by supporting the analysis and development of meaningful mathematical connections.

Future studies could complement participant observation with non-participant observation or stimulated-recall interviews to examine how these connections emerge under different data-collection conditions.

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Ethical statement

This study was approved by the Scientific Ethics Committee of Universidad Católica de la Santísima Concepción, Chile (ORD. No. 02/2026). The research protocol, informed consent form, institutional authorization letter, and data collection documents were reviewed and approved by the Committee. The study ensured the confidentiality and anonymity of the participants, and informed consent was obtained in accordance with the approved ethical procedures. The research was approved on January 16, 2026, and subsequently ratified in an expedited session.

Competing interests

All authors declare that they have no competing interests.

Author contributions

C.A.R-N: conceptualization, theoretical and methodological design, data collection, data analysis and interpretation, and preparation of the initial draft of the manuscript. J.A.C-G: data analysis and interpretation, identification and characterization of mathematical connections, and critical review of the manuscript. M. K.G-L: data analysis and interpretation, methodological review, strengthening of the mathematics education perspective, adaptation of the manuscript to publication requirements, and critical review of the manuscript. S.S.: data analysis and interpretation, critical review of the manuscript, and contribution to the discussion of the findings. D.D.C-M: data analysis and interpretation, methodological review, and critical review of the manuscript. A.B.: conceptualization, theoretical grounding from the Onto-Semiotic Approach, analysis and interpretation of results, academic supervision, and critical review of the manuscript. I.R.M-B: data analysis and interpretation, organization of empirical evidence, and critical review of the manuscript. All authors reviewed and approved the final version of the manuscript.

Data availability

The data supporting the findings of this study are available from the authors upon reasonable request. Requests may be addressed to the corresponding author via email.

AI disclosure

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APPENDIX

Table 2

Constitution of the mathematical connection from an onto-semiotic view.

Problem situation: Explain and establish relationships between exponential and logarithmic functions through their definitions, properties, procedures, representations, and inverse relationship.				
Code and Mathematical practice	Mathematical process(es)	Primary objects (cognitive configuration)	Semiotic function (Expression ↔ Content)	ETC mathematical connection
C1. Define the exponential function	Definition; communication; representation–meaning	Language: verbal and symbolic expression ($f(x) = a^x$). Concepts/definitions: function, exponential function, base, exponent. Propositions: conditions ($a > 0$), ($a \neq 1$). Procedures: identification of the components of the expression. Arguments: explanation of the conditions defining the exponential function.	Symbolic/verbal definition ↔ institutional meaning of exponential function	Meaning
	Definition; communication;	Language: verbal and symbolic logarithmic notation. Concepts/definitions: logarithm,	Symbolic/verbal definition ↔ institutional	Meaning

C2. Define the logarithmic function	representation–meaning	logarithmic function, base, argument, exponent. Propositions: conditions on the base and argument. Procedures: identification and interpretation of components. Arguments: explanation of conditions of existence.	meaning of logarithmic function	
C3. Explain logarithmation	Communication; definition; enunciation; representation–meaning	Language: verbal and symbolic logarithmic/exponential expressions. Concepts: logarithm, exponentiation, exponent. Propositions: logarithmation determines the exponent associated with a power. Procedures: conversion between logarithmic and exponential forms. Arguments: justification of their conceptual correspondence.	Logarithmic expression $\leftrightarrow$ exponentiation $\leftrightarrow$ meaning of the exponent	Meaning
C4. Identify structural elements	Definition; analysis; particularization	Language: base, exponent, argument, variable. Concepts: exponential/logarithmic function and their components. Propositions: structural restrictions of these components. Procedures: identification and differentiation of elements. Arguments: relationships among components and functional structure.	Components $\leftrightarrow$ structure of the function	Meaning / Feature
C5. Determine domain and range	Enunciation; analysis; generalization–particularization	Language: interval, set, inequality and graphical notation. Concepts: domain, range, variable, function. Propositions: restrictions defining admissible values. Procedures: solving restrictions/inequalities and identifying value sets. Arguments: justification of domain and range from function conditions.	Conditions/restrictions $\leftrightarrow$ domain and range	Feature
C6. Describe functional behavior	Communication; enunciation; analysis; generalization	Language: verbal, numerical and graphical. Concepts: growth, decrease, variation, function. Propositions: behavior according to the value of the base. Procedures: reading and comparing tables/graphs. Arguments: explanation of variation based on function properties.	Graph/table $\leftrightarrow$ functional behavior	Feature
C7. Apply algebraic properties	Procedures; enunciation; argumentation; generalization–particularization	Language: algebraic and symbolic expressions. Concepts: exponential/logarithmic properties and equivalence. Propositions: algebraic laws. Procedures: transformation and simplification of expressions. Arguments: justification of the validity of transformations.	Algebraic property $\leftrightarrow$ valid transformation	Feature / Procedural
C8. Substitute values	Procedures; particularization	Language: symbolic and numerical. Concepts: variable, function, image. Propositions: functional correspondence.	General expression + input $\leftrightarrow$ particular output	Procedural

		Procedures: substitution and numerical calculation. Arguments: verification of the obtained value when present.		
C9. Construct a table of values	Procedures; particularization; materialization–representation; synthesis	Language: symbolic, numerical and tabular. Concepts: variable, ordered pair, function. Propositions: functional correspondence between inputs and outputs. Procedures: substitution, calculation and organization of values. Arguments: consistency of calculated pairs with the function.	Expression $\leftrightarrow$ numerical values $\leftrightarrow$ table	Procedural / Representations
C10. Graph the function	Procedures; materialization–representation; synthesis	Language: symbolic, tabular and graphical. Concepts: Cartesian plane, ordered pair, graph, function. Propositions: points satisfy the function. Procedures: locate points and construct the curve. Arguments: consistency between expression, points and graph.	Algebraic/table representation $\leftrightarrow$ graphical representation	Representations / Part–whole
C11. Translate between representations	Communication; materialization–representation–meaning; analysis–synthesis	Language: verbal, symbolic, numerical, tabular and graphical. Concepts: function and representation. Propositions: representations preserve the same mathematical object/meaning. Procedures: conversion and coordination among registers. Arguments: explanation of the correspondence among representations.	Representation A $\leftrightarrow$ Representation B $\leftrightarrow$ same mathematical meaning	Different representations
C12. Identify asymptotes	Enunciation; analysis; idealization; particularization	Language: graphical, symbolic and verbal. Concepts: asymptote, domain, limit behavior, function. Propositions: the graph approaches a line under specific conditions. Procedures: identify restrictions and interpret graphical behavior. Arguments: justify the asymptote from the function structure.	Restriction/behavior $\leftrightarrow$ asymptote $\leftrightarrow$ global graph	Part–whole
C13. Identify key points	Procedures; particularization; synthesis; materialization–representation	Language: numerical, symbolic and graphical. Concepts: point, ordered pair, graph. Propositions: selected points belong to the function. Procedures: calculate and locate coordinates. Arguments: use of points to justify the global graphical form.	Particular points $\leftrightarrow$ global graphical configuration	Part–whole / Representations
C14. Activate prior knowledge	Communication; personalization–institutionalization; representation–meaning	Language: verbal and symbolic. Concepts: powers, exponents, functions and previously learned objects. Propositions: previously institutionalized properties. Procedures: recalled operations and rules. Arguments: use of previous knowledge to support new meanings.	Prior mathematical knowledge $\leftrightarrow$ new mathematical meaning	Instruction-oriented

C15. Sequence mathematical content	Communication; procedures; analysis–synthesis; institutionalization	Language: instructional and mathematical discourse. Concepts: mathematical objects involved in the sequence. Propositions: relations establishing dependency among stages. Procedures: ordered sequence of mathematical actions. Arguments: rationale for the instructional progression when explicit.	Ordered practices $\leftrightarrow$ instructional organization	Instruction-oriented
C16. Recapitulate previous results	Communication; synthesis; personalization–institutionalization	Language: verbal/symbolic. Concepts: previously constructed mathematical objects. Propositions: established results and properties. Procedures: recovery and reuse of previous results. Arguments: connection between previous and current meanings.	Previous result/meaning $\leftrightarrow$ current mathematical explanation	Instruction-oriented
C17. Recognize inverse functions	Enunciation; analysis; representation–meaning	Language: verbal and symbolic. Concepts: exponential, logarithmic and inverse functions. Propositions: exponential and logarithmic functions are inverses. Procedures: recognition/comparison of functional forms. Arguments: explanation of inverse correspondence when present.	Exponential function $\leftrightarrow$ logarithmic function	Reversibility
C18. Interchange variables	Procedures; analysis–synthesis; materialization–representation	Language: symbolic. Concepts: variable and inverse function. Propositions: inverse correspondence interchanges dependent/independent variables. Procedures: $(x \Leftrightarrow y)$. Arguments: justification of variable exchange as part of inversion.	$(x \Leftrightarrow y) \leftrightarrow$ inverse transformation	Procedural / Reversibility
C19. Solve for the variable	Procedures; analysis–synthesis; argumentation	Language: symbolic/algebraic. Concepts: equation, equivalence and inverse function. Propositions: algebraic equivalence of transformed expressions. Procedures: algebraic isolation of the variable. Arguments: validity of each equivalent transformation.	Algebraic transformation $\leftrightarrow$ equivalent inverse expression	Procedural / Reversibility
C20. Verify inverted ordered pairs	Procedures; argumentation; generalization–particularization	Language: numerical, tabular and graphical. Concepts: ordered pair and inverse function. Propositions: $((x, y) \Leftrightarrow (y, x))$. Procedures: calculate, interchange and verify coordinates. Arguments: verification that exchanged pairs belong to inverse functions.	$((x, y) \Leftrightarrow (y, x)) \leftrightarrow$ inverse correspondence	Reversibility
C21. Relate domain and range of inverse functions	Enunciation; analysis–synthesis; generalization	Language: symbolic, set and verbal. Concepts: domain, range and inverse function. Propositions: $(Dom(f) = Ran(f^{-1}))$ and $(Ran(f) = Dom(f^{-1}))$. Procedures:	Domain $\leftrightarrow$ range under inversion	Reversibility

		comparison of domains/ranges. Arguments: justification from inverse correspondence.		
C22. Compare graphical/struc- tural properties	Analysis–synthesis; argumentation; generalization; representation– meaning	Language: graphical, symbolic and verbal. Concepts: exponential/logarithmic functions, asymptotes, symmetry, inverse function. Propositions: corresponding properties under inversion. Procedures: compare graphs and structural properties. Arguments: justification of the correspondence between both functions.	Structural properties of $(f) \leftrightarrow$ corresponding properties of (f^{-1})	Reversibility / Part– whole
C23. Establish “if...then...” reasoning	Enunciation; argumentation; analysis; generalization– particularization	Language: verbal and symbolic conditional statements. Concepts: mathematical condition, consequence and functional relation. Propositions: premise– conclusion statements. Procedures: deductive derivation when required. Arguments: logical justification connecting premise and consequence.	Mathematical condition $\rightarrow$ mathematical consequence	Implication

Source: Authors' own elaboration.